

Chapter 5

STRUCTURAL ANALYSIS PROCEDURES

5.1 GENERAL

5.1.1 Scope. This chapter provides minimum requirements for the structural analysis procedures prescribed in Sec. 4.4.1. If the alternate design procedure of Alternative Simplified Chapter 4 is used, this chapter does not apply.

5.1.2 Definitions

Base: See Sec. 4.1.3.

Base shear: See Sec. 4.1.3.

Building: See Sec. 4.1.3.

Component: See Sec. 1.1.4.

Dead load: See Sec. 4.1.3.

Design earthquake ground motion: See Sec. 1.1.4.

Design strength: See Sec. 4.1.3.

Diaphragm: See Sec. 4.1.3.

Eccentrically braced frame (EBF): See Sec. 4.1.3.

Inverted pendulum-type structures: See Sec. 4.1.3.

Live load: See Sec. 4.1.3.

Maximum considered earthquake ground motion: See Sec. 3.1.3.

Moment frame: See Sec. 4.1.3.

Nominal strength: See Sec. 4.1.3.

Occupancy importance factor: See Sec. 1.1.4.

Partition: A nonstructural interior wall that spans from floor to ceiling, to the floor or roof structure immediately above, or to subsidiary structural members attached to the structure above.

P-delta effect: The secondary effect on shears and moments of structural members induced due to displacement of the structure.

Registered design professional: See Sec. 2.1.3.

Required strength: See Sec. 4.1.3.

Seismic Design Category: See Sec. 1.1.4.

Seismic-force-resisting system: See Sec. 1.1.4.

Seismic forces: See Sec. 1.1.4.

Seismic response coefficient: Coefficient C_S as determined in Sec. 5.2.2.1.

Shear wall: See Sec. 4.1.3.

Story: See Sec. 4.1.3.

Story shear: The summation of design lateral forces at levels above the story under consideration.

Structure: See Sec. 1.1.4.

5.1.3 Notation

A_o	The area of the load-carrying foundation.
A_B	The base area of the structure.
A_i	The area of shear wall i .
A_x	The torsional amplification factor.
a_d	The incremental factor related to P -delta effects in Sec. 5.2.6.2.
C_d	See Sec. 4.1.4.
C_r	The approximate period coefficient given in Sec. 5.2.2.1.
C_S	The seismic response coefficient.
C_{Sm}	The modal seismic response coefficient determined in Sec. 5.3.4.
C_u	Coefficient for upper limit on calculated period given in Table 5.2-1.
C_{vx}	The vertical distribution factor given in Sec. 5.2.3.
C_{vxm}	The vertical distribution factor in the m th mode given in Sec. 5.3.5.
C_w	The effective shear wall area coefficient defined in Sec. 5.2.2.1.
D_s	The total depth of the stratum in Eq. 5.6-10.
F_i	See Sec. 4.1.4.
F_x	See Sec. 1.1.5.
F_{xm}	The portion of the seismic base shear, V_m , induced at a Level x as determined in Sec. 5.3.5.
G	γ_s^2/g = the average shear modulus for the soils beneath the foundation at large strain levels.
G_o	γ_{so}^2/g = the average shear modulus for the soils beneath the foundation at small strain levels.
h_i	The height above the base to Level i .
h_n	The height above the base to the highest level of the structure.
h_x	The height above the base to Level x .
\bar{h}	The effective height of the structure as defined in Sec. 5.6.2.1.1.
I	See Sec. 1.1.5.
I_o	The static moment of inertia of the load-carrying foundation, see Sec. 5.6.2.1.1.
k	The distribution exponent given in Sec. 5.2.3.
\bar{k}	The stiffness of the fixed-base structure as defined in Sec. 5.6.2.1.1.
K_y	The lateral stiffness of the foundation as defined in Sec. 5.6.2.1.1.
K_θ	The rocking stiffness of the foundation as defined in Sec. 5.6.2.1.1.
L_i	The length of shear wall i .
L_o	The overall length of the side of the foundation in the direction being analyzed, Sec. 5.6.2.1.2.
M_o, M_{o1}	The overturning moment at the foundation-soil interface as determined in Sec. 5.2.5 and 5.3.6.

M_t	The torsional moment resulting from the location of the building masses, Sec. 5.2.4.1.
M_{ta}	The accidental torsional moment as determined in Sec. 5.2.4.2.
m	A subscript denoting the mode of vibration under consideration; i.e., $m=1$ for the fundamental mode.
N	Number of stories.
P_x	The total unfactored vertical design load at and above level x .
Q_E	See Sec. 4.1.4.
R	See Sec. 4.1.4.
r_a	The characteristic foundation length defined by Eq. 5.6-7.
r_m	The characteristic foundation length as defined by Eq. 5.6-8.
S_I	See Sec. 3.1.4.
S_{am}	The design spectral response acceleration at the period corresponding to mode m .
S_{DI}	See Sec. 3.1.4.
S_{DS}	See Sec. 3.1.4.
T	See Sec. 4.1.4.
\tilde{T}	The effective period of the flexibly supported structure as determined by Eq. 5.6-3.
T_a	The approximate fundamental period of the building as determined in Sec. 5.2.2.1.
T_m	The period of the m^{th} mode of vibration of the structure in the direction of interest.
V	The total design shear at the base of the structure in the direction of interest, as determined using the procedure of Sec. 5.2, including Sec. 5.2.1.
V_I	The portion of seismic base shear, V , contributed by the fundamental mode.
V_t	The design value of the seismic base shear as determined in Sec. 5.3.7.
V_x	The seismic design shear in Story x .
\tilde{V}	The total design base shear including consideration of soil-structure interaction.
ΔV	The reduction in V as determined in Sec. 5.6.2.1.
ΔV_I	The reduction of V_I as determined in Sec. 5.6.3.1.
v_s	See Sec. 3.1.4.
v_{so}	The average shear wave velocity for the soils beneath the foundation at small strain levels.
W	See Sec. 1.1.5.
\bar{W}	The effective seismic weight as defined in Sec. 5.6.2.1 and 5.6.3.1.
\bar{W}_m	The effective seismic weight of the m^{th} mode of vibration of the structure determined in accordance with Eq. 5.3-2.
w_i	See Sec. 4.1.4.
w_x	See Sec. 1.1.5.
Level x	See Sec. 1.1.5.
α	The relative weight density of the structure and the soil as determined in Sec. 5.6.2.1.1.

α_θ	The dynamic foundation stiffness modifier for rocking (see <i>Commentary</i>).
$\tilde{\beta}$	The fraction of critical damping for the coupled structure-foundation system, determined in Sec. 5.6.2.1.2.
β_o	The foundation damping factor as specified in Sec. 5.6.2.1.2.
γ	The average unit weight of soil.
γ	The member inelastic deformations.
Δ	See Sec. 4.1.4.
$\tilde{\Delta}_m$	The modal story drift including the effects of soil-structure interaction.
δ_{avg}	The average of the displacements at the extreme points of the structure at Level x .
δ_{max}	The maximum displacement at Level x .
δ_{xm}	The modal deflection of Level x at the center of the mass at and above Level x , as determined by Eq. 5.3-8.
$\bar{\delta}_{x1}$	The modified modal deflections (for the first mode) as determined by Eq. 5.6-14.
$\bar{\delta}_{xm}$	The modified modal deflections (for mode m) as determined by Eq. 5.6-15.
θ	The stability coefficient for P -delta effects as determined in Sec. 5.2.6.2.
θ_{max}	The maximum permitted stability coefficient as determined by Eq. 5.2-17.
τ	The overturning moment reduction factor.
ϕ	The strength reduction, capacity reduction, or resistance factor.
Ω_0	See Sec. 4.1.4.

5.2 EQUIVALENT LATERAL FORCE PROCEDURE

An equivalent lateral force analysis shall consist of the application of equivalent static lateral forces to a linear mathematical model of the structure. The lateral forces applied in each direction shall sum to a total seismic base shear given by Sec. 5.2.1 and shall be distributed vertically in accordance with Sec. 5.2.3. For purposes of analysis, the structure shall be considered fixed at the base.

5.2.1 Seismic base shear. The seismic base shear, V , in a given direction shall be determined in accordance with the following equation:

$$V = C_s W \quad (5.2-1)$$

where:

C_s = the seismic response coefficient determined in accordance with Sec. 5.2.1.1 and

W = the total dead load and applicable portions of other loads listed below:

1. In areas used for storage, a minimum of 25 percent of the floor live load shall be applicable. Floor live load in public garages and open parking structures is not applicable.
2. Where an allowance for partition load is included in the floor load design, the actual partition weight or a minimum weight of 10 psf (0.500 kN/m²) of floor area, whichever is greater, shall be applicable.
3. Total operating weight of permanent equipment.

4. In areas where the design flat roof snow load does not exceed 30 pounds per square foot, the effective snow load is permitted to be taken as zero. In areas where the design snow load is greater than 30 psf (1.4 kN/m²), where siting and load duration conditions warrant, and where approved by the authority having jurisdiction, the effective snow load is permitted to be reduced to not less than 20 percent of the design snow load.

5.2.1.1 Calculation of seismic response coefficient. The seismic response coefficient, C_s , shall be determined in accordance with the following equation:

$$C_s = \frac{S_{DS}}{R/I} \quad (5.2-2)$$

where:

S_{DS} = the design spectral response acceleration parameter in the short period range as determined from Sec. 3.3.3,

R = the response modification factor from Table 4.3-1, and

I = the occupancy importance factor determined in accordance with Sec. 1.3.

The value of the seismic response coefficient computed in accordance with Eq. 5.2-2 need not exceed the following:

$$C_s = \frac{S_{D1}}{T(R/I)} \text{ for } T \leq T_L \quad (5.2-3)$$

$$C_s = \frac{S_{D1}T_L}{T^2(R/I)} \text{ for } T > T_L \quad (5.2-4)$$

where R and I are as defined above and

S_{D1} = the design spectral response acceleration parameter at a period of 1.0 second as determined from Sec. 3.3.3,

T = the fundamental period of the structure (in seconds) determined in Sec. 5.2.2, and

T_L = Long-period transition period (in seconds) determined in Sec. 3.3.1

C_s shall not be taken less than 0.01.

For buildings and structures located where S_I is equal to or greater than 0.6g, C_s shall not be taken less than:

$$C_s = \frac{0.5S_I}{R/I} \quad (5.2-5)$$

where R and I are as defined above and

S_I = the mapped maximum considered earthquake spectral response acceleration parameter determined in accordance with Sec. 3.3.1.

For regular structures 5 stories or less in height and having a period, T , of 0.5 seconds or less, the seismic response coefficient, C_s , shall be permitted to be calculated using values of 1.5 and 0.6, respectively, for the mapped maximum considered earthquake spectral response acceleration parameters S_S and S_I .

A soil-structure interaction reduction is permitted where determined using Sec. 5.6 or other generally accepted procedures approved by the authority having jurisdiction.

5.2.2 Period determination. The fundamental period of the building, T , in the direction under consideration shall be established using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. The fundamental period, T , so calculated, shall not exceed the product of C_u , from Table 5.2-1, and T_a , calculated in accordance with Sec. 5.2.2.1. As an alternative to performing an analysis to determine the fundamental period of the structure, T , it shall be permitted to use the approximate period equations of Sec. 5.2.2.1 directly.

Table 5.2-1 Coefficient for Upper Limit on Calculated Period

Value of S_{DI} ^a	C_u
$S_{DI} \geq 0.4$	1.4
$S_{DI} = 0.3$	1.4
$S_{DI} = 0.2$	1.5
$S_{DI} = 0.15$	1.6
$S_{DI} \leq 0.1$	1.7

Note: ^a Use straight line interpolation for intermediate values of S_{DI} .

5.2.2.1 Approximate fundamental period. The approximate fundamental period, T_a , in seconds, shall be determined from the following equation:

$$T_a = C_r h_n^x \quad (5.2-6)$$

where h_n is the height in feet (meters) above the base to the highest level of the structure and the values of C_r and x shall be determined from Table 5.2-2.

Table 5.2-2 Values of Approximate Period Parameters C_r and x

Structure Type	C_r	x
Moment resisting frame systems of steel in which the frames resist 100 percent of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting where subjected to seismic forces.	0.028 (metric 0.0724)	0.8
Moment resisting frame systems of reinforced concrete in which the frames resist 100 percent of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting where subjected to seismic forces.	0.016 (metric 0.0466)	0.9
Eccentrically braced steel frames and buckling restrained braced frames	0.03 (metric 0.0731)	0.75
All other structural systems	0.02 (metric 0.0488)	0.75

Alternatively, the approximate fundamental period, T_a , in seconds, is permitted to be determined from the following equation for concrete and steel moment resisting frame structures not exceeding 12 stories in height and having a minimum story height of not less than 10 ft (3 m):

$$T_a = 0.1N \quad (5.2-7)$$

where N = number of stories.

The approximate fundamental period, T_a , in seconds, for masonry or concrete shear wall structures is permitted to be determined from the following equation:

$$T_a = \frac{0.0019}{\sqrt{C_w}} h_n \quad (5.2-8)$$

where C_w is a coefficient related to the effective shear wall area and h_n is as defined above. The metric equivalent of Eq. 5.2-8 is:

$$T_a = \frac{0.0062}{\sqrt{C_w}} h_n$$

The coefficient C_w shall be calculated from the following equation:

$$C_w = \frac{100}{A_B} \sum_{i=1}^n \left(\frac{h_n}{h_i} \right)^2 \frac{A_i}{\left[1 + 0.83 \left(\frac{h_i}{L_i} \right)^2 \right]} \quad (5.2-9)$$

where:

A_B = the base area of the structure,

A_i = the area of shear wall i ,

L_i = the length of shear wall i ,

h_n = the height above the base to the highest level of the structure,

h_i = the height of shear wall i , and

n = the number of shear walls in the building effective in resisting lateral forces in the direction under consideration.

5.2.3 Vertical distribution of seismic forces. The lateral force, F_x , induced at any level shall be determined from the following equations:

$$F_x = C_{vx} V \quad (5.2-10)$$

and

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^n w_i h_i^k} \quad (5.2-11)$$

where:

C_{vx} = vertical distribution factor,

V = total design lateral force or shear at the base of the structure,

w_i and w_x = the portion of the total gravity load of the structure, W , located or assigned to Level i or x ,

- h_i and h_x = the height from the base to Level i or x , and
- k = an exponent related to the effective fundamental period of the structure as follows:
- For structures having a period of 0.5 seconds or less, $k = 1$
- For structures having a period of 2.5 seconds or more, $k = 2$
- For structures having a period between 0.5 and 2.5 seconds, k shall be determined by linear interpolation between 1 and 2 or may be taken equal to 2.

5.2.4 Horizontal shear distribution. The seismic design story shear in any story, V_x , shall be determined from the following equation:

$$V_x = \sum_{i=x}^n F_i \quad (5.2-12)$$

where F_i = the portion of the seismic base shear, V , induced at Level i .

The seismic design story shear, V_x , shall be distributed to the various vertical elements of the seismic-force-resisting system in the story under consideration based on the relative lateral stiffnesses of the vertical resisting elements and the diaphragm.

5.2.4.1 Inherent torsion. The distribution of lateral forces at each level shall consider the effect of the inherent torsional moment, M_t , resulting from eccentricity between the locations of the center of mass and the center of rigidity.

5.2.4.2 Accidental torsion. In addition to the inherent torsional moment, the distribution of lateral forces also shall include accidental torsional moments, M_{ta} , caused by an assumed displacement of the mass each way from its actual location by a distance equal to 5 percent of the dimension of the structure perpendicular to the direction of the applied forces.

5.2.4.3 Dynamic amplification of torsion. For structures assigned to Seismic Design Category C, D, E, or F, where Type 1 torsional irregularity exists as defined in Table 4.3-2, the effects of torsional irregularity shall be accounted for by multiplying the sum of M_t plus M_{ta} at each level by a torsional amplification factor, A_x , determined from the following equation:

$$A_x = \left(\frac{\delta_{max}}{1.2\delta_{avg}} \right)^2 \quad (5.2-13)$$

where:

- δ_{max} = the maximum displacement at Level x , and
- δ_{avg} = the average of the displacements at the extreme points of the structure at Level x .

The torsional amplification factor, A_x , is not required to exceed 3.0. The more severe loading for each element shall be considered for design.

5.2.5 Overturning. The structure shall be designed to resist overturning effects caused by the seismic forces determined in Sec. 5.2.3. At any story, the increment of overturning moment in the story under consideration shall be distributed to the various vertical force resisting elements in the same proportion as the distribution of the horizontal shears to those elements.

The overturning moments at Level x , M_x , shall be determined from Eq. 5.2-14 as follows:

$$M_x = \sum_{i=x}^n F_i (h_i - h_x) \quad (5.2-14)$$

where:

- F_i = the portion of the seismic base shear, V , induced at Level i , and
 h_i and h_x = the height from the base to Level i or x .

The foundations of structures, except inverted pendulum-type structures, shall be permitted to be designed for three-fourths of the foundation overturning design moment, M_0 , determined using Eq. 5.2-14 at the foundation-soil interface.

5.2.6 Drift determination and P-delta limit. Story drifts shall be determined in accordance with this section. Determination of story drifts shall be based on the application of the design seismic forces to a mathematical model of the physical structure. The model shall include the stiffness and strength of all elements that are significant to the distribution of forces and deformations in the structure and shall represent the spatial distribution of the mass and stiffness of the structure. In addition, the model shall comply with the following:

1. Stiffness properties of reinforced concrete and masonry elements shall consider the effects of cracked sections and
2. For steel moment resisting frame systems, the contribution of panel zone deformations to overall story drift shall be included.

5.2.6.1 Story drift determination. The design story drift, Δ , shall be computed as the difference of the deflections at the center of mass at the top and bottom of the story under consideration.

Exception: For structures of Seismic Design Category C, D, E or F having plan irregularity Type 1a or 1b of Table 4.3-2, the design story drift, Δ , shall be computed as the largest difference of the deflections along any of the edges of the structure at the top and bottom of the story under consideration.

The deflections of Level x , δ_x , shall be determined in accordance with following equation:

$$\delta_x = \frac{C_d \delta_{xe}}{I} \quad (5.2-15)$$

where:

- C_d = the deflection amplification factor from Table 4.3-1,
 δ_{xe} = the deflections determined by an elastic analysis, and
 I = the occupancy importance factor determined in accordance with Sec. 1.3.

The elastic analysis of the seismic-force-resisting system shall be made using the prescribed seismic design forces of Sec. 5.2.3.

For determining compliance with the story drift limits of Sec. 4.5.1, it shall be permitted to determine the elastic drifts, δ_{xe} , using seismic design forces based on the computed fundamental period of the structure without the upper limit ($C_u T_u$) specified in Sec. 5.2.2.

Where nonlinear analysis is required by Sec. 5.2.6.2 and the nonlinear static procedure is used, the design story drift, Δ , shall be determined according to Sec. A5.2.4.

5.2.6.2 P-delta limit. stability coefficient, θ , as determined for each level of the structure by the following equation, shall not exceed 0.10:

$$\theta = \frac{P_x \Delta I}{V_x h_{sx} C_d} \quad (5.2-16)$$

where:

- P_x = the total vertical design load at and above Level x . Where calculating the vertical design load for purposes of determining P-delta effects, the individual load factors need not exceed 1.0.
- Δ = the design story drift calculated in accordance with Sec. 5.2.6.1.
- I = the occupancy importance factor determined in accordance with Sec.1.3
- V_x = the seismic shear force acting between Level x and $x - 1$.
- h_{sx} = the story height below Level x .
- C_d = the deflection amplification factor from Table 4.3-1.

Exception: The stability coefficient θ , shall be permitted to exceed 0.10 if the resistance to lateral forces is determined to increase continuously in a monotonic nonlinear static (pushover) analysis to the target displacement as determined in Sec. A5.2.3. P-delta effects shall be included in the analysis.

5.3 RESPONSE SPECTRUM PROCEDURE

A modal response spectrum analysis shall consist of the analysis of a linear mathematical model of the structure to determine the maximum accelerations, forces, and displacements resulting from the dynamic response to ground shaking represented by the design response spectrum. The analysis shall be performed in accordance with the requirements of this section. For purposes of analysis, the structure shall be permitted to be considered to be fixed at the base or, alternatively, it shall be permitted to use realistic assumptions with regard to the stiffness of foundations. The symbols used in this section have the same meaning as those for similar terms used in Sec. 5.2 but with the subscript m denoting quantities relating to the m^{th} mode.

5.3.1 Modeling. A mathematical model of the structure shall be constructed that represents the spatial distribution of mass and stiffness throughout the structure. For regular structures with independent orthogonal seismic-force-resisting systems, independent two-dimensional models are permitted to be constructed to represent each system. For irregular structures or structures without independent orthogonal systems, a three-dimensional model incorporating a minimum of three dynamic degrees of freedom consisting of translation in two orthogonal plan directions and torsional rotation about the vertical axis shall be included at each level of the structure. Where the diaphragms are not rigid compared to the vertical elements of the lateral-force-resisting system, the model should include representation of the diaphragm's flexibility and such additional dynamic degrees of freedom as are required to account for the participation of the diaphragm in the structure's dynamic response. In addition, the model shall comply with the following:

1. Stiffness properties of concrete and masonry elements shall consider the effects of cracked sections and
2. The contribution of panel zone deformations to overall story drift shall be included for steel moment frame resisting systems.

5.3.2 Modes. An analysis shall be conducted to determine the natural modes of vibration for the structure including the period of each mode, the modal shape vector ϕ , the modal participation factor, and modal mass. The analysis shall include a sufficient number of modes to obtain a combined modal mass participation of at least 90 percent of the actual mass in each of two orthogonal directions.

5.3.3 Modal properties. The required periods, mode shapes, and participation factors of the structure shall be calculated by established methods of structural analysis for the fixed-base condition using the masses and elastic stiffnesses of the seismic-force-resisting system.

5.3.4 Modal base shear. The portion of the base shear contributed by the m^{th} mode, V_m , shall be determined from the following equations:

$$V_m = C_{sm} \bar{W}_m \quad (5.3-1)$$

$$\bar{W}_m = \frac{\left(\sum_{i=1}^n w_i \phi_{im} \right)^2}{\sum_{i=1}^n w_i \phi_{im}^2} \quad (5.3-2)$$

where:

- C_{sm} = the modal seismic response coefficient as determined in this section,
 \bar{W}_m = the effective modal gravity load including portions of the live load as defined in Sec. 5.2.1,
 w_i = the portion of the total gravity load of the structure at Level i , and
 ϕ_{im} = the displacement amplitude at the i^{th} level of the structure where vibrating in its m^{th} mode.

The modal seismic response coefficient, C_{sm} , shall be determined in accordance with the following equation:

$$C_{sm} = \frac{S_{am}}{R/I} \quad (5.3-3)$$

where:

- S_{am} = The design spectral response acceleration at period T_m determined from either Sec. 3.3.4 or Sec. 3.4.4,
 R = the response modification factor determined from Table 4.3-1,
 I = the occupancy importance factor determined in accordance with Sec. 1.3, and
 T_m = the modal period of vibration (in seconds) of the m^{th} mode of the structure.

Exceptions:

1. Where the standard design response spectrum of Sec. 3.3.4 is used for structures on Site Class D, E or F soils, the modal seismic design coefficient, C_{sm} , for modes other than the fundamental mode that have periods less than 0.3 seconds is permitted to be determined by the following equation:

$$C_{sm} = \frac{0.4 S_{DS}}{R/I} (1 + 5T_m) \quad (5.3-4)$$

where S_{DS} is as defined in Sec. 3.3.3 and R , I , and T_m are as defined above.

2. Where the standard design response spectrum of Sec. 3.3.4 is used for structures where any modal period of vibration, T_m , exceeds T_L , the modal seismic design coefficient, C_{sm} , for that mode is permitted to be determined by the following equation:

$$C_{sm} = \frac{S_{DI} T_L}{(R/I) T_m^2} \quad (5.3-5)$$

where R , I , and T_m are as defined above and S_{DI} is the design spectral response acceleration parameter at a period of 1 second as determined in Sec. 3.3.3. and T_L is the Long-period transition period as defined in Sec. 3.3.4.

The reduction due to soil-structure interaction as determined in Sec. 5.6.3 shall be permitted to be used.

5.3.5 Modal forces, deflections, and drifts. The modal force, F_{xm} , at each level shall be determined by the following equations:

$$F_{xm} = C_{vsm} V_m \quad (5.3-6)$$

and

$$C_{vsm} = \frac{w_x \phi_{xm}}{\sum_{i=1}^n w_i \phi_{im}} \quad (5.3-7)$$

where:

- C_{vsm} = the vertical distribution factor in the m^{th} mode,
- V_m = the total design lateral force or shear at the base in the m^{th} mode,
- w_i, w_x = the portion of the total gravity load, W , located or assigned to Level i or x ,
- ϕ_{xm} = the displacement amplitude at the x^{th} level of the structure where vibrating in its m^{th} mode, and
- ϕ_{im} = the displacement amplitude at the i^{th} level of the structure where vibrating in its m^{th} mode.

The modal deflection at each level, δ_{xm} , shall be determined by the following equations:

$$\delta_{xm} = \frac{C_d \delta_{xem}}{I} \quad (5.3-8)$$

and

$$\delta_{xem} = \left(\frac{g}{4\pi^2} \right) \left(\frac{T_m^2 F_{xm}}{w_x} \right) \quad (5.3-9)$$

where:

- C_d = the deflection amplification factor determined from Table 4.3-1,
- δ_{xem} = the deflection of Level x in the m^{th} mode at the center of the mass at Level x determined by an elastic analysis,
- g = the acceleration due to gravity,
- I = the occupancy importance factor determined in accordance with Sec. 1.3,
- T_m = the modal period of vibration, in seconds, of the m^{th} mode of the structure,
- F_{xm} = the portion of the seismic base shear in the m^{th} mode, induced at Level x , and
- w_x = the portion of the total gravity load of the structure, W , located or assigned to Level x .

The modal drift in a story, Δ_m , shall be computed as the difference of the deflections, δ_{xm} , at the top and bottom of the story under consideration.

5.3.6 Modal story shears and moments. The story shears, story overturning moments, and the shear forces and overturning moments in vertical elements of the structural system at each level due to the seismic forces determined from the appropriate equation in Sec. 5.3.5 shall be computed for each mode by linear static methods.

5.3.7 Design values. The design value for the modal base shear, V_i ; each of the story shear, moment, and drift quantities; and the deflection at each level shall be determined by combining their modal

values as obtained from Sec. 5.3.5 and 5.3.6. The combination shall be carried out by taking the square root of the sum of the squares of each of the modal values or by the complete quadratic combination technique. The complete quadratic combination shall be used where closely spaced periods in the translational and torsional modes will result in cross-correlation of the modes.

A base shear, V , shall be calculated using the equivalent lateral force procedure in Sec. 5.2. For the purpose of this calculation, the fundamental period of the structure, T , in seconds, shall not exceed the coefficient for upper limit on the calculated period, C_u , times the approximate fundamental period of the structure, T_a . Where the design value for the modal base shear, V_i , is less than 85 percent of the calculated base shear, V , using the equivalent lateral force procedure, the design story shears, moments, drifts, and floor deflections shall be multiplied by the following modification factor:

$$0.85 \frac{V}{V_i} \quad (5.3-10)$$

where:

V = the equivalent lateral force procedure base shear calculated in accordance with this section and Sec. 5.2 and

V_i = the modal base shear calculated in accordance with this section.

Where soil-structure interaction is considered in accordance with Sec. 5.6, the value of V may be taken as the reduced value of V .

5.3.8 Horizontal shear distribution. The distribution of horizontal shear shall be in accordance with the requirements of Sec. 5.2.4 except that amplification of torsion per Sec. 5.2.4.3 is not required for that portion of the torsion included in the dynamic analysis model.

5.3.9 Foundation overturning. The foundation overturning moment at the foundation-soil interface shall be permitted to be reduced by 10 percent.

5.3.10 P-delta effects. The P-delta effects shall be determined in accordance with Sec. 5.2.6. The story drifts and story shears shall be determined in accordance with Sec. 5.2.6.1.

5.4 LINEAR RESPONSE HISTORY PROCEDURE

A linear response history analysis shall consist of an analysis of a linear mathematical model of the structure to determine its response, through methods of numerical integration, to suites of ground motion acceleration histories compatible with the design response spectrum for the site. The analysis shall be performed in accordance with the provisions of this section. For the purposes of analysis, the structure shall be permitted to be considered to be fixed at the base or, alternatively, it shall be permitted to use realistic assumptions with regard to the stiffness of foundations.

5.4.1 Modeling. Mathematical models shall conform to the requirements of Sec. 5.3.1.

5.4.2 Ground motion. A suite of not fewer than three appropriate ground motions shall be used in the analysis. Ground motion shall conform to the requirements of this section.

5.4.2.1 Two-dimensional analysis. Where two-dimensional analyses are performed, each ground motion shall consist of a horizontal acceleration history selected from an actual recorded event. Appropriate acceleration histories shall be obtained from records of events having magnitudes, fault distance, and source mechanisms that are consistent with those that control the maximum considered earthquake. Where the required number of appropriate recorded ground motion records are not available, appropriate simulated ground motion records shall be used to make up the total number required. The ground motions shall be scaled such that for each period between $0.2T$ and $1.5T$ (where T is the natural period of the structure in the fundamental mode for the direction of response being analyzed) the average of the five-percent-damped response spectra for the suite of motions is not less

than the corresponding ordinate of the design response spectrum, determined in accordance with Sec. 3.3.4 or 3.4.4.

5.4.2.2 Three-dimensional analysis. Where three-dimensional analysis is performed, ground motions shall consist of pairs of appropriate horizontal ground motion acceleration components that shall be selected and scaled from individual recorded events. Appropriate ground motions shall be selected from events having magnitudes, fault distance, and source mechanisms that are consistent with those that control the maximum considered earthquake. Where the required number of recorded ground motion pairs are not available, appropriate simulated ground motion pairs shall be used to make up the total number required. For each pair of horizontal ground motion components, an SRSS spectrum shall be constructed by taking the square root of the sum of the squares of the five-percent-damped response spectra for the components (where an identical scale factor is applied to both components of a pair). Each pair of motions shall be scaled such that for each period between $0.2T$ and $1.5T$ (where T is the natural period of the fundamental mode of the structure) the average of the SRSS spectra from all horizontal component pairs is not less than 1.3 times the corresponding ordinate of the design response spectrum, determined in accordance with Sec. 3.3.4 or 3.4.4.

5.4.3 Response parameters. For each ground motion analyzed, the individual response parameters shall be scaled by the quantity I/R where I is the occupancy importance factor determined in accordance with Sec. 1.3 and R is the response modification coefficient selected in accordance with Sec. 4.3-1. The maximum value of the base shear, V_i , member forces, Q_{Ei} , and the interstory drifts, δ_i , at each story scaled as indicated above shall be determined. Where the maximum scaled base shear predicted by the analysis, V_i , is less than that given by Eq. 5.2-4 or, in Seismic Design Categories E and F, Eq. 5.2-5, the scaled member forces, Q_{Ei} , shall be additionally scaled by the factor V/V_i where V is the minimum base shear determined in accordance with Eq. 5.2-4 or, for structures in Seismic Design Category E or F, Eq. 5.2-5.

If at least seven ground motions are analyzed, the design member forces, Q_E , used in the load combinations of Sec. 4.2.2 and the design interstory drift, Δ , used in the evaluation of drift in accordance with Sec. 4.5.1 shall be permitted to be taken, respectively, as the average of the scaled Q_{Ei} and δ_i values determined from the analyses and scaled as indicated above. If fewer than seven ground motions are analyzed, the design member forces, Q_E , and the design interstory drift, Δ , shall be taken as the maximum value of the scaled Q_{Ei} and δ_i values determined from the analyses.

Where these *Provisions* require the consideration of the seismic load effect with overstrength as defined in Sec. 4.2.2.2, the value of $\Omega_0 Q_E$ need not be taken larger than the maximum of the unscaled value, Q_{Ei} , obtained from the suite of analyses.

5.5 NONLINEAR RESPONSE HISTORY PROCEDURE

A nonlinear response history analysis shall consist of an analysis of a mathematical model of the structure that directly accounts for the nonlinear hysteretic behavior of the structure's components to determine its response, through methods of numerical integration, to suites of ground motion acceleration histories compatible with the design response spectrum for the site. The analysis shall be performed in accordance with the requirements of this section.

5.5.1 Modeling. A mathematical model of the structure shall be constructed that represents the spatial distribution of mass throughout the structure. The hysteretic behavior of elements shall be modeled consistent with suitable laboratory test data and shall account for all significant yielding, strength degradation, stiffness degradation, and hysteretic pinching indicated by such test data. Strength of elements shall be based on expected values considering material overstrength, strain hardening, and hysteretic strength degradation. Linear properties, consistent with the provisions of Section 5.3.1 shall be permitted to be used for those elements demonstrated by the analysis to remain within their linear range of response. The structure shall be assumed to have a fixed base or, alternatively, it shall be

permitted to use realistic assumptions with regard to the stiffness and load carrying characteristics of the foundations consistent with site-specific soils data and rational principles of engineering mechanics.

For regular structures with independent orthogonal seismic-force-resisting systems, independent two-dimensional models shall be permitted to be constructed to represent each system. For structures having plan irregularity Type 1a, 1b, 4, or 5 of Table 4.3-2 or structures without independent orthogonal systems, a three-dimensional model incorporating a minimum of three dynamic degrees of freedom consisting of translation in two orthogonal plan directions and torsional rotation about the vertical axis at each level of the structure shall be used. Where the diaphragms are not rigid compared to the vertical elements of the lateral-force-resisting system, the model shall include representation of the diaphragm's flexibility and such additional dynamic degrees of freedom as are required to account for the participation of the diaphragm in the structure's dynamic response.

5.5.2 Ground motion and other loading. Ground motion shall conform to the requirements of Sec. 5.4.2. The structure shall be analyzed for the effects of these ground motions simultaneously with the effects of dead load in combination with not less than 25 percent of the required live loads.

5.5.3 Response parameters. For each ground motion analyzed, individual response parameters consisting of the maximum value of the individual member forces, Q_{Ei} , member inelastic deformations, γ_i , and story drifts, Δ_i , shall be determined.

If at least seven ground motions are analyzed, the design values of member forces, Q_E , member inelastic deformations, γ , and story drift, Δ , shall be permitted to be taken, respectively, as the average of the scaled Q_{Ei} , γ_i , and Δ_i values determined from the analyses. If fewer than seven ground motions are analyzed, the design member forces, Q_E , design member inelastic deformations, γ , and the design story drift, Δ , shall be taken as the maximum value of the scaled Q_{Ei} , γ_i , and Δ_i values determined from the analyses.

5.5.3.1 Member strength. The adequacy of members to resist the load combinations of Sec 4.2.2 need not be evaluated.

Exception: Where the *Provisions* require the consideration of the seismic load effect with overstrength, determined in accordance with Sec. 4.2.2.2, the maximum value of Q_{Ei} obtained from the suite of analyses shall be taken in place of the quantity $\Omega_0 Q_E$.

5.5.3.2 Member deformation. The adequacy of individual members and their connections to withstand the design deformations, γ , predicted by the analyses shall be evaluated based on laboratory test data for similar components. The effects of gravity and other loads on member deformation capacity shall be considered in these evaluations. Member deformation shall not exceed two thirds of the lesser of: the value that results in loss of ability to carry gravity loads or the value at which member strength has deteriorated to less than the 67 percent of the peak strength.

5.5.3.3 Story drift. The design story drifts, Δ , obtained from the analyses shall not exceed 125 percent of the drift limit specified in Sec. 4.5.1.

5.5.4 Design review. A review of the design of the seismic-force-resisting system and the supporting structural analyses shall be performed by an independent team consisting of registered design professionals in the appropriate disciplines and others with experience in seismic analysis methods and the theory and application of nonlinear seismic analysis and structural behavior under extreme cyclic loads. The design review shall include, but need not be limited to, the following:

1. Review of any site-specific seismic criteria employed in the analysis including the development of site-specific spectra and ground motion time histories,
2. Review of acceptance criteria used to demonstrate the adequacy of structural elements and systems to withstand the calculated force and deformation demands, together with laboratory and other data used to substantiate such criteria,

3. Review of the preliminary design including the determination of the target displacement of the structure and the margins remaining beyond these displacements, and
4. Review of the final design of the entire structural system and all supporting analyses.

5.6 SOIL-STRUCTURE INTERACTION EFFECTS

5.6.1 General. The requirements set forth in this section are permitted to be used to incorporate the effects of soil-structure interaction in the determination of the design earthquake forces and the corresponding displacements of the structure when the model used for structural response analysis does not directly incorporate the effects of foundation flexibility (i.e., the model corresponds to a fixed-base condition with no foundation springs). The use of these requirements will decrease the design values of the base shear, lateral forces, and overturning moments but may increase the computed values of the lateral displacements and the secondary forces associated with the P-delta effects.

The requirements for use with the equivalent lateral force procedure are given in Sec. 5.6.2 and those for use with the response spectrum procedure are given in Sec. 5.6.3. The provisions in Sec. 5.6 shall not be used if a flexible-base, rather than a fixed base, foundation is directly modeled in the structural response analysis.

5.6.2 Equivalent lateral force procedure. The following requirements are supplementary to those presented in Sec. 5.2.

5.6.2.1 Base shear. To account for the effects of soil-structure interaction, the base shear, V , determined from Eq. 5.2-1 may be reduced to:

$$\tilde{V} = V - \Delta V \quad (5.6-1)$$

where the reduction, ΔV , shall be computed as follows:

$$\Delta V = \left[C_s - \tilde{C}_s \left(\frac{0.05}{\tilde{\beta}} \right)^{0.4} \right] \bar{W} \quad (5.6-2)$$

where:

- C_s = the seismic response coefficient computed from Eq. 5.2-2 using the fundamental natural period of the fixed-base structure as specified in Sec. 5.2.2,
- \tilde{C}_s = the seismic response coefficient computed from Eq. 5.2-2 using the effective period of the flexibly supported structure defined in Sec. 5.6.2.1.1,
- $\tilde{\beta}$ = the fraction of critical damping for the structure-foundation system determined in Sec. 5.6.2.1.2, and
- \bar{W} = the effective gravity load of the structure, which shall be taken as $0.7W$, except that for structures where the gravity load is concentrated at a single level, it shall be taken equal to W .

The reduced base shear, \tilde{V} , shall in no case be taken less than $0.7V$.

5.6.2.1.1 Effective building period. The effective period of the flexibly supported structure, \tilde{T} , shall be determined as follows:

$$\tilde{T} = T \sqrt{1 + \frac{\bar{k}}{K_y} \left(1 + \frac{K_y \bar{h}^2}{K_\theta} \right)} \quad (5.6-3)$$

where:

T = the fundamental period of the structure as determined in Sec. 5.2.2;

\bar{k} = the stiffness of the fixed-base structure, defined by the following:

$$\bar{k} = 4\pi^2 \left(\frac{\bar{W}}{gT^2} \right) \quad (5.6-4)$$

\bar{h} = the effective height of the structure which shall be taken as 0.7 times the total height, h_n , except that for structures where the gravity load is effectively concentrated at a single level, it shall be taken as the height to that level;

K_y = the lateral stiffness of the foundation defined as the horizontal force at the level of the foundation necessary to produce a unit deflection at that level, the force and the deflection being measured in the direction in which the structure is analyzed;

K_θ = the rocking stiffness of the foundation defined as the moment necessary to produce a unit average rotation of the foundation, the moment and rotation being measured in the direction in which the structure is analyzed; and

g = the acceleration due to gravity.

The foundation stiffnesses, K_y and K_θ , shall be computed by established principles of foundation mechanics (see the *Commentary*) using soil properties that are compatible with the soil strain levels associated with the design earthquake motion. The average shear modulus, G , for the soils beneath the foundation at large strain levels and the associated shear wave velocity, v_s , needed in these computations shall be determined from Table 5.6-1 where:

v_{so} = the average shear wave velocity for the soils beneath the foundation at small strain levels (10^{-3} percent or less),

$G_o = \gamma v_{so}^2 / g$ = the average shear modulus for the soils beneath the foundation at small strain levels, and

γ = the average unit weight of the soils.

Table 5.6-1 Values of G/G_o and v_s/v_{so}

	$S_{DS}/2.5$			
	≤ 0.10	0.15	0.20	≥ 0.30
Value of G/G_o	0.81	0.64	0.49	0.42
Value of v_s/v_{so}	0.90	0.80	0.70	0.65

Alternatively, for structures supported on mat foundations that rest at or near the ground surface or are embedded in such a way that the side wall contact with the soil cannot be considered to remain effective during the design ground motion, the effective period of the structure may be determined from:

$$\tilde{T} = T \sqrt{1 + \frac{25\alpha}{v_s^2 T^2} \frac{r_a \bar{h}}{\alpha_\theta r_m^3} \left(1 + \frac{1.12 r_a \bar{h}^2}{\alpha_\theta r_m^3} \right)} \quad (5.6-5)$$

where:

α = the relative weight density of the structure and the soil, defined by:

$$\alpha = \frac{\bar{W}}{\gamma A_0 \bar{h}} \quad (5.6-6)$$

r_a and r_m = characteristic foundation lengths, defined by:

$$r_a = \sqrt{\frac{A_0}{\pi}} \quad (5.6-7)$$

and

$$r_m = \sqrt[4]{\frac{4I_0}{\pi}} \quad (5.6-8)$$

where:

A_o = the area of the foundation,

I_o = the static moment of the foundation about a horizontal centroidal axis normal to the direction in which the structure is analyzed, and

α_θ = dynamic foundation stiffness modifier for rocking (see *Commentary*).

5.6.2.1.2 Effective damping. The effective damping factor for the structure-foundation system, $\tilde{\beta}$, shall be computed as follows:

$$\tilde{\beta} = \beta_0 + \frac{0.05}{\left(\frac{\tilde{T}}{T}\right)^3} \quad (5.6-9)$$

where β_0 = the foundation damping factor as specified in Figure 5.6-1.

For values of $S_{DS}/2.5$ between 0.10 and 0.20, values of β_0 shall be determined by linear interpolation between the solid lines and the dashed lines of Figure 5.6-1.

The quantity r in Figure 5.6-1 is a characteristic foundation length that shall be determined as follows:

$$\text{For } \frac{\bar{h}}{L_0} \leq 0.5, r = r_a$$

$$\text{For } \frac{\bar{h}}{L_0} \geq 1.0, r = r_m$$

For intermediate values of $\frac{\bar{h}}{L_0}$, r shall be determined by linear interpolation.

where:

L_0 = the overall length of the side of the foundation in the direction being analyzed, and

r_a and r_m = characteristic foundation lengths, defined in Sec. 5.6.2.1.1.

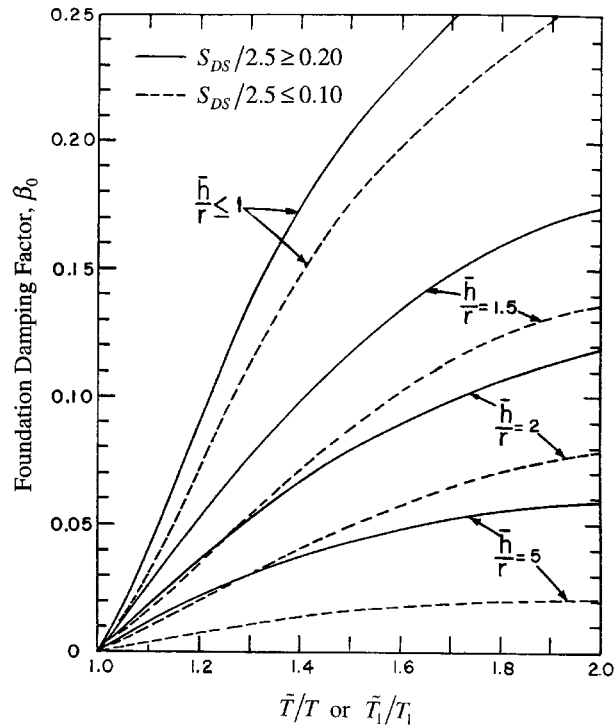


Figure 5.6-1 Foundation Damping Factor

Exception: For structures supported on point bearing piles and in all other cases where the foundation soil consists of a soft stratum of reasonably uniform properties underlain by a much stiffer, rock-like deposit with an abrupt increase in stiffness, the factor β_0 in Eq. 5.6-9 shall be replaced by:

$$\beta'_0 = \left(\frac{4D_s}{v_s \bar{T}} \right)^2 \beta_0 \quad (5.6-10)$$

if $\frac{4D_s}{v_s \bar{T}} < 1$, where D_s is the total depth of the stratum.

The value of $\tilde{\beta}$ computed from Eq. 5.6-9, with or without the adjustment represented by Eq. 5.6-10, shall in no case be taken less than 0.05 or greater than 0.20.

5.6.2.2 Vertical distribution of seismic forces. The distribution over the height of the structure of the reduced total seismic force, \tilde{V} , shall be considered to be the same as for the fixed-base structure.

5.6.2.3 Other effects. The modified story shears, overturning moments, and torsional effects about a vertical axis shall be determined as for structures without interaction using the reduced lateral forces.

The modified deflections, $\tilde{\delta}_x$, shall be determined as follows:

$$\tilde{\delta}_x = \frac{\tilde{V}}{V} \left(\frac{M_0 h_x}{K_\theta} + \delta_x \right) \quad (5.6-11)$$

where:

- M_o = the overturning moment at the base determined in accordance with Sec. 5.2.5 using the unmodified seismic forces and not including the reduction permitted in the design of the foundation,
- h_x = the height above the base to the level under consideration, and
- δ_x = the deflections of the fixed-base structure as determined in Sec. 5.2.6.1 using the unmodified seismic forces.

The modified story drifts and P-delta effects shall be evaluated in accordance with the requirements of Sec. 5.2.6 using the modified story shears and deflections determined in this section.

5.6.3 Response spectrum procedure. The following requirements are supplementary to those presented in Sec. 5.3.

5.6.3.1 Modal base shears. To account for the effects of soil-structure interaction, the base shear corresponding to the fundamental mode of vibration, V_1 , is permitted to be reduced to:

$$\tilde{V}_1 = V_1 - \Delta V_1 \quad (5.6-12)$$

The reduction, ΔV_1 , shall be computed in accordance with Eq. 5.6-2 with \bar{W} taken as equal to the gravity load \bar{W}_1 defined by Eq. 5.3-2, C_s computed from Eq. 5.3-3 using the fundamental period of the fixed-base structure, T_1 , and \tilde{C}_s computed from Eq. 5.3-3 using the fundamental period of the flexibly supported structure, \tilde{T}_1 .

The period \tilde{T}_1 shall be determined from Eq. 5.6-3, or from Eq. 5.6-5 where applicable, taking $T = \tilde{T}_1$, evaluating \bar{k} from Eq. 5.6-4 with $\bar{W} = \bar{W}_1$, and computing \bar{h} as follows:

$$\bar{h} = \frac{\sum_{i=1}^n w_i \phi_{i1} h_i}{\sum_{i=1}^n w_i \phi_{i1}} \quad (5.6-13)$$

The above designated values of \bar{W} , \bar{h} , T , and \tilde{T} also shall be used to evaluate the factor α from Eq. 5.6-6 and the factor β_o from Figure 5.6-1. No reduction shall be made in the shear components contributed by the higher modes of vibration. The reduced base shear, \tilde{V}_1 , shall in no case be taken less than $0.7V_1$.

5.6.3.2 Other modal effects. The modified modal seismic forces, story shears, and overturning moments shall be determined as for structures without interaction using the modified base shear, \tilde{V}_1 , instead of V_1 . The modified modal deflections, $\tilde{\delta}_{xm}$, shall be determined as follows:

$$\tilde{\delta}_{x1} = \frac{\tilde{V}_1}{V_1} \left(\frac{M_o h_x}{K_\theta} + \delta_{x1} \right) \quad (5.6-14)$$

and

$$\tilde{\delta}_{xm} = \delta_{xm} \text{ for } m = 2, 3, \dots \quad (5.6-15)$$

where:

- M_{o1} = the overturning base moment for the fundamental mode of the fixed-base structure, as determined in Sec. 5.3.6 using the unmodified modal base shear V_1 , and

δ_{xm} = the modal deflections at Level x of the fixed-base structure as determined in Sec. 5.3.5 using the unmodified modal shears, V_m .

The modified modal drift in a story, $\tilde{\Delta}_m$, shall be computed as the difference of the deflections, $\tilde{\delta}_{xm}$, at the top and bottom of the story under consideration.

5.6.3.3 Design values The design values of the modified shears, moments, deflections, and story drifts shall be determined as for structures without interaction by taking the square root of the sum of the squares of the respective modal contributions. In the design of the foundation, the overturning moment at the foundation-soil interface determined in this manner may be reduced by 10 percent as for structures without interaction.

The effects of torsion about a vertical axis shall be evaluated in accordance with the requirements of Sec. 5.2.4 and the P-delta effects shall be evaluated in accordance with the requirements of Sec. 5.2.6.1, using the story shears and drifts determined in Sec. 5.6.3.2.

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