

Chapter 15

STRUCTURES WITH DAMPING SYSTEMS

15.1 GENERAL

15.1.1 Scope. Every structure with a damping system and every portion thereof shall be designed and constructed in accordance with the requirements of these *Provisions* as modified by this Chapter. Where damping devices are used across the isolation interface of a seismically isolated structure, displacements, velocities, and acceleration shall be determined in accordance with Chapter 13.

15.1.2 Definitions

Base: See Sec. 4.1.3.

Base shear: See Sec. 4.1.3.

Component: See Sec. 1.1.4.

Damping device: A flexible structural element of the damping system that dissipates energy due to relative motion of each end of the device. Damping devices include all pins, bolts, gusset plates, brace extensions, and other components required to connect damping devices to the other elements of the structure. Damping devices may be classified as either displacement-dependent or velocity-dependent, or a combination thereof, and may be configured to act in either a linear or nonlinear manner.

Damping system: The collection of structural elements that includes all the individual damping devices, all structural elements or bracing required to transfer forces from damping devices to the base of the structure, and the structural elements required to transfer forces from damping devices to the seismic-force-resisting system.

Design displacement: See Sec. 13.1.2.

Design earthquake ground motion: See Sec. 1.1.4.

Displacement-dependent damping device: The force response of a displacement-dependent damping device is primarily a function of the relative displacement, between each end of the device. The response is substantially independent of the relative velocity between each of the device, and/or the excitation frequency.

Maximum displacement: See Sec. 13.1.2.

Maximum considered earthquake ground motion: See Sec. 3.1.3.

Registered design professional: See Sec. 2.1.3.

Seismic Design Category: See Sec. 1.1.4.

Seismic-force-resisting system: See Sec. 1.1.4.

Seismic forces: See Sec. 1.1.4.

Seismic response coefficient: See Sec. 5.1.2.

Site Class: See Sec. 3.1.3.

Structure: See Sec. 1.1.4.

Total design displacement: See Sec. 13.1.2.

Total maximum displacement: See Sec. 13.1.2.

Velocity-dependent damping device: The force-displacement relation for a velocity-dependent damping device is primarily a function of the relative velocity between each end of the device, and may also be a function of the relative displacement between each end of the device.

15.1.3 Notation

B_{ID}	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mI} ($m=1$) and period of structure equal to T_{ID} .
B_{IE}	Numerical coefficient as set forth in Table 15.6-1 for the effective damping equal to $\beta_I + \beta_{VI}$ and period equal to T_I .
B_{IM}	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mM} ($m=1$) and period of structure equal to T_{IM} .
B_{mD}	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mI} and period of structure equal to T_m .
B_{mM}	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mM} and period of structure equal to T_m .
B_R	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_R and the period of structure equal to T_R .
B_{V+I}	Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to the sum of viscous damping in the fundamental mode of vibration of the structure in the direction of interest, β_{Vm} ($m = 1$), plus inherent damping, β_I , and period of structure equal to T_I .
C_d	See Sec. 4.1.4.
C_{mFD}	Force coefficient as set forth in Table 15.7-1.
C_{mFV}	Force coefficient as set forth in Table 15.7-2.
C_{SI}	Seismic response coefficient of the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.4.2.4 or Sec. 15.5.2.4 ($m = 1$).
C_{Sm}	Seismic response coefficient of the m^{th} mode of vibration of the structure in the direction of interest, Sec. 15.4.2.4 ($m = 1$) or Sec. 15.4.2.6 ($m > 1$).
C_{SR}	Seismic response coefficient of the residual mode of vibration of the structure in the direction of interest, Sec. 15.5.2.8.
D_{ID}	Fundamental mode design displacement at the center rigidity of the roof level of structure in the direction under consideration, Sec. 15.5.3.2.
D_{IM}	Fundamental mode maximum displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.5.
D_{mD}	Design displacement at the center of rigidity of the roof level of the structure due to the m^{th} mode of vibration in the direction under consideration, Sec. 15.4.3.2.
D_{mM}	Maximum displacement at the center of rigidity of the roof level of the structure due to the m^{th} mode of vibration in the direction under consideration, Sec. 15.4.3.5.
D_{RD}	Residual mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.2.
D_{RM}	Residual mode maximum displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.5.
D_Y	Displacement at the center of rigidity of the roof level of the structure at the effective yield point of the seismic-force-resisting system, Sec. 15.6.3.

E_{loop}	See Sec. 13.1.3.
f_i	Lateral force at Level i of the structure distributed approximately in accordance with Sec. 5.2.3, Sec. 15.5.2.3.
F_{iI}	Inertial force at Level i (or mass point i) in the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.5.2.9.
F_{im}	Inertial force at Level i (or mass point i) in the m^{th} mode of vibration of the structure in the direction of interest, Sec. 15.4.2.7.
F_{iR}	Inertial force at Level i (or mass point i) in the residual mode of vibration of the structure in the direction of interest, Sec. 15.5.2.9.
h_i	See Sec. 5.1.3.
h_r	Height of the structure above the base to the roof level, Sec. 15.5.2.3.
I	See Sec. 1.1.5.
q_H	Hysteresis loop adjustment factor as determined in Sec. 15.6.2.2.1.
m	See Sec. 5.1.3.
Q_{DSD}	Force in an element of the damping system required to resist design seismic forces of displacement-dependent damping devices, Sec. 15.7.3.3.
Q_E	See Sec. 4.1.4.
Q_{mDSV}	Forces in an element of the damping system required to resist design seismic forces of velocity-dependent damping devices due to the m^{th} mode of vibration of structure in the direction of interest, Sec. 15.7.3.3.
Q_{mSFRS}	Force in a element of the damping system equal to the design seismic force of the m^{th} mode of vibration of the seismic force resisting system in the direction of interest, 15.7.3.3.
R	See Sec. 4.1.4.
S_I	See Sec. 3.1.4.
S_{DI}	See Sec. 3.1.4.
S_{DS}	See Sec. 3.1.4.
S_{MI}	See Sec. 3.1.4.
S_{MS}	See Sec. 3.1.4.
T_0	See Sec. 3.1.4.
T_I	See Sec. 15.5.2.3.
T_{ID}	Effective period, in seconds, of the fundamental mode of vibration of the structure at the design displacement in the direction under consideration, as prescribed by Sec. 15.4.2.5 or Sec. 15.5.2.5.
T_{IM}	Effective period, in seconds, of the fundamental mode of vibration of the structure at the maximum displacement in the direction under consideration, as prescribed by Sec. 15.4.2.5 or Sec. 15.5.2.5.
T_m	See Sec. 5.1.3.
T_R	Period, in seconds, of the residual mode of vibration of the structure in the direction under consideration, Sec. 15.5.2.7.
T_S	See Sec. 3.1.4.

V	See Sec. 5.1.3.
V_m	Design value of the seismic base shear of the m^{th} mode of vibration of the structure in the direction of interest, Sec. 5.3.4 or Sec. 15.4.2.2.
V_{min}	Minimum allowable value of base shear permitted for design of the seismic-force-resisting system of the structure in the direction of interest, Sec. 15.2.2.1.
V_R	Design value of the seismic base shear of the residual mode of vibration of the structure in a given direction, as determined in Sec. 15.5.2.6.
W	See Sec. 1.1.5.
\bar{W}_I	Effective fundamental mode seismic weight determined in accordance with Eq. 5.3-2 for $m = 1$.
\bar{W}_R	Effective residual mode seismic weight determined in accordance with Eq. 15.5-13.
w_i	See Sec. 4.1.4.
w_x	See Sec. 1.1.5.
α	Velocity exponent relating damping device force to damping device velocity.
β_{mD}	Total effective damping of the m^{th} mode of vibration of the structure in the direction of interest at the design displacement, Sec. 15.6.2.
β_{mM}	Total effective damping of the m^{th} mode of vibration of the structure in the direction of interest at the maximum displacement, Sec. 15.6.2.
β_{HD}	Component of effective damping of the structure in the direction of interest due to post-yield hysteric behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand μ_D , Sec. 15.6.2.2.
β_{HM}	Component of effective damping of the structure in the direction of interest due to post-yield hysteric behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand, μ_M , Sec. 15.6.2.2.
β_I	Component of effective damping of the structure due to the inherent dissipation of energy by elements of the structure, at or just below the effective yield displacement of the seismic-force-resisting system, Sec. 15.6.2.1.
β_R	Total effective damping in the residual mode of vibration of the structure in the direction of interest, calculated in accordance with Sec. 15.6.2 ($\mu_D = 1.0$ and $\mu_M = 1.0$).
β_{Vm}	Component of effective damping of the m^{th} mode of vibration of the structure in the direction of interest due to viscous dissipation of energy by the damping system, at or just below the effective yield displacement of the seismic-force-resisting system, Sec. 15.6.2.3.
δ_i	Elastic deflection of Level i of the structure due to applied lateral force, f_i , Sec. 15.5.2.3.
δ_{iD}	Fundamental mode design earthquake deflection of Level i at the center of rigidity of the structure in the direction under consideration, Sec. 15.5.3.1.
δ_D	Total design earthquake deflection of Level i at the center of rigidity of the structure in the direction under consideration, Sec. 15.5.3.
δ_M	Total maximum earthquake deflection of Level i at the center of rigidity of the structure in the direction under consideration, Sec. 15.5.3.
δ_{RD}	Residual mode design earthquake deflection of Level i at the center of rigidity of the structure in the direction under consideration, Sec. 15.5.3.

δ_{im}	Deflection of Level i in the m^{th} mode of vibration at the center of rigidity of the structure in the direction under consideration, Sec. 15.6.2.3.
Δ_{ID}	Design earthquake story drift due to the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.5.3.3.
Δ_D	Total design earthquake story drift of the structure in the direction of interest, Sec. 15.5.3.3.
Δ_M	Total maximum earthquake story drift of the structure in the direction of interest, Sec. 15.5.3.
Δ_{mD}	Design earthquake story drift due to the m^{th} mode of vibration of the structure in the direction of interest, Sec. 15.4.3.3.
Δ_{RD}	Design earthquake story drift due to the residual mode of vibration of the structure in the direction of interest, Sec. 15.5.3.3.
μ	Effective ductility demand on the seismic-force-resisting system in the direction of interest.
μ_D	Effective ductility demand on the seismic-force-resisting system in the direction of interest due to the design earthquake, Sec. 15.6.3.
μ_M	Effective ductility demand on the seismic-force-resisting system in the direction of interest due to the maximum considered earthquake, Sec. 15.6.3.
μ_{max}	Maximum allowable effective ductility demand on the seismic-force-resisting system due to design earthquake, Sec. 15.6.4.
ρ	See Sec. 4.1.4.
ϕ_{i1}	Displacement amplitude at Level i of the fundamental mode of vibration of the structure in the direction of interest, normalized to unity at the roof level, Sec. 15.5.2.3.
ϕ_{iR}	Displacement amplitude at Level i of the residual mode of vibration of the structure in the direction of interest normalized to unity at the roof level, Sec. 15.5.2.7.
Γ_1	Participation factor of fundamental mode of vibration of the structure in the direction of interest, Sec. 15.4.2.3 or Sec. 15.5.2.3 ($m = 1$).
Γ_m	Participation factor on the m^{th} mode of vibration of the structure in the direction of interest, Sec. 15.4.2.3.
Γ_R	Participation factor of the residual mode of vibration of the structure in the direction of interest, Sec. 15.5.2.7.
Ω_0	See Sec. 4.1.4.
∇_{ID}	Design earthquake story velocity due to the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.5.3.4.
∇_D	Total design earthquake story velocity of the structure in the direction of interest, Sec. 15.4.3.4.
∇_M	Total maximum earthquake story velocity of the structure in the direction of interest, Sec. 15.5.3.
∇_{mD}	Design earthquake story velocity due to the m^{th} mode of vibration of the structure in the direction of interest, Sec. 15.4.3.4.

15.2 GENERAL DESIGN REQUIREMENTS

15.2.1 Seismic Design Category A. Seismic Design Category A structures with a damping system shall be designed using the design spectral response acceleration determined in accordance with Sec. 3.3.3 and the analysis methods and design provisions required for Seismic Design Category B structures.

15.2.2 System requirements. Design of the structure shall consider the basic requirements for the seismic-force-resisting system and the damping system as defined in the following sections. The seismic-force-resisting system shall have the required strength to meet the forces defined in Section 15.2.2.1. The combination of the seismic-force-resisting system and the damping system may be used to meet the drift requirement.

15.2.2.1 Seismic-force-resisting system. Structures that contain a damping system are required to have a basic seismic-force-resisting system that, in each lateral direction, conforms to one of the types indicated in Table 4.3-1.

The design of the seismic-force-resisting system in each direction shall satisfy the requirements of Sec. 15.7 and the following:

1. The seismic base shear used for design of the seismic-force-resisting system shall not be less than V_{min} , where V_{min} is determined as the greater of the values computed using Eq. 15.2-1 and 15.2-2 as follows:

$$V_{min} = \frac{V}{B_{V+1}} \quad (15.2-1)$$

$$V_{min} = 0.75V \quad (15.2-2)$$

where:

V = seismic base shear in the direction of interest, determined in accordance with Sec. 5.2,

B_{V+1} = numerical coefficient as set forth in Table 15.6-1 for effective damping equal to the sum of viscous damping in the fundamental mode of vibration of the structure in the direction of interest, $\beta_{Vm} (m = 1)$, plus inherent damping, β_i , and period of structure equal to T_l .

Exception: The seismic base shear used for design of the seismic-force-resisting system shall not be taken as less than $1.0V$, if either of the following conditions apply:

- a. In the direction of interest, the damping system has less than two damping devices on each floor level, configured to resist torsion.
 - b. The seismic-force-resisting system has plan irregularity Type 1b (Table 4.3-2) or vertical irregularity Type 1b (Table 4.3-3).
2. Minimum strength requirements for elements of the seismic-force-resisting system that are also elements of the damping system or are otherwise required to resist forces from damping devices shall meet the additional requirements of Sec. 15.7.2.

15.2.2.2 Damping system. Elements of the damping system shall be designed to remain elastic for design loads including unreduced seismic forces of damping devices as required in Sec. 15.7.2, unless it is shown by analysis or test that inelastic response of elements would not adversely affect damping system function and inelastic response is limited in accordance with the requirements of Sec. 15.7.2.4.

15.2.3 Ground motion

15.2.3.1 Design spectra. Spectra for the design earthquake and the maximum considered earthquake developed in accordance with Sec. 13.2.3.1 shall be used for the design and analysis of all structures with a damping system. Site-specific design spectra shall be developed and used for design of all structures with a damping system if either of the following conditions apply:

1. The structure is located on a Class F site or
2. The structure is located at a site with S_I greater than 0.6.

15.2.3.2 Time histories. Ground-motion time histories for the design earthquake and the maximum considered earthquake developed in accordance with Sec. 13.2.3.2 shall be used for design and analysis of all structures with a damping system if either of the following conditions apply:

1. The structure is located at a site with S_I greater than 0.6.
2. The damping system is explicitly modeled and analyzed using the time history analysis method.

15.2.4 Procedure selection

All structures with a damping system shall be designed using linear procedures, nonlinear procedures, or a combination of linear and nonlinear procedures, as permitted in this section.

Regardless of the analysis method used, the peak dynamic response of the structure and elements of the damping system shall be confirmed by using the nonlinear response history procedure if the structure is located at a site with S_I greater than 0.6.

15.2.4.1 Nonlinear procedures. The nonlinear procedures of Sec. 15.3 are permitted to be used for design of all structures with damping systems.

15.2.4.2 Response spectrum procedure. The response spectrum procedure of Sec. 15.4 is permitted to be used for design of structures with damping systems provided that:

1. In the direction of interest, the damping system has at least two damping devices in each story, configured to resist torsion; and
2. The total effective damping of the fundamental mode, $\beta_{mD}(m = 1)$, of the structure in the direction of interest is not greater than 35 percent of critical.

15.2.4.3 Equivalent lateral force procedure. The equivalent lateral force procedure of Sec. 15.5 is permitted to be used for design of structures with damping systems provided that:

1. In the direction of interest, the damping system has at least two damping devices in each story, configured to resist torsion;
2. The total effective damping of the fundamental mode, $\beta_{mD}(m = 1)$, of the structure in the direction of interest is not greater than 35 percent of critical;
3. The seismic-force-resisting system does not have plan irregularity Type 1a or 1b (Table 4.3-2) or vertical irregularity Type 1a, 1b, 2, or 3 (Table 4.3-3);
4. Floor diaphragms are rigid as defined in Sec. 4.3.2.1; and
5. The height of the structure above the base does not exceed 100 ft (30 m).

15.2.5 Damping system

15.2.5.1 Device design. The design, construction, and installation of damping devices shall be based on maximum earthquake response and the following conditions:

1. Low-cycle, large-displacement degradation due to seismic loads;
2. High-cycle, small-displacement degradation due to wind, thermal, or other cyclic loads;
3. Forces or displacements due to gravity loads;
4. Adhesion of device parts due to corrosion or abrasion, biodegradation, moisture, or chemical exposure; and
5. Exposure to environmental conditions, including but not limited to temperature, humidity, moisture, radiation (e.g., ultraviolet light), and reactive or corrosive substances (e.g., salt water).

Damping devices subject to failure by low-cycle fatigue shall resist wind forces without slip, movement, or inelastic cycling.

The design of damping devices shall incorporate the range of thermal conditions, device wear, manufacturing tolerances, and other effects that cause device properties to vary during the design life of the device.

15.2.5.2 Multi-axis movement. Connection points of damping devices shall provide sufficient articulation to accommodate simultaneous longitudinal, lateral, and vertical displacements of the damping system.

15.2.5.3 Inspection and periodic testing. Means of access for inspection and removal of all damping devices shall be provided.

The registered design professional responsible for design of the structure shall establish an appropriate inspection and testing schedule for each type of damping device to ensure that the devices respond in a dependable manner throughout the design life. The degree of inspection and testing shall reflect the established in-service history of the damping devices, and the likelihood of change in properties over the design life of devices.

15.2.5.4 Quality control. As part of the quality assurance plan developed in accordance with Sec. 2.2.1, the registered design professional responsible for the structural design shall establish a quality control plan for the manufacture of damping devices. As a minimum, this plan shall include the testing requirements of Sec. 15.9.2.

15.3 NONLINEAR PROCEDURES

The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 15.9. The nonlinear force-deflection characteristics of damping devices shall be modeled, as required, to explicitly account for device dependence on frequency, amplitude, and duration of seismic loading.

15.3.1 Nonlinear response history procedure. A nonlinear response history (time history) analysis shall utilize a mathematical model of the structure and the damping system as provided in Sec. 5.5.1 and this section. The model shall directly account for the nonlinear hysteretic behavior of elements of the structure and the damping devices to determine its response, through methods of numerical integration, to suites of ground motions compatible with the design response spectrum for the site.

The analysis shall be performed in accordance with Sec. 5.5 together with the requirements of this section. Inherent damping of the structure shall not be taken greater than five percent of the critical unless test data consistent with the levels of deformation at or just below the effective yield displacement of the seismic-force-resisting system support higher values.

If the calculated force in the element of the seismic-force-resisting system does not exceed 1.5 times its nominal strength, that element may be modeled as linear.

15.3.1.1 Damping device modeling. Mathematical models of displacement-dependent damping devices shall include the hysteretic behavior of the devices consistent with test data and accounting for all significant changes in strength, stiffness, and hysteretic loop shape. Mathematical models of velocity-dependent damping devices shall include the velocity coefficient consistent with test data. If this coefficient changes with time and/or temperature, such behavior shall be modeled explicitly. The elements of damping devices connecting damper units to the structure shall be included in the model.

Exception: If the properties of the damping devices are expected to change during the duration of the time history analysis, the dynamic response may be enveloped by the upper and lower limits of device properties. All these limit cases for variable device properties must satisfy the same conditions as if the time dependent behavior of the devices were explicitly modeled.

15.3.1.2 Response parameters. In addition to the response parameters given in Sec. 5.5.3 for each ground motion analyzed, individual response parameters consisting of the maximum value of the discrete damping device forces, displacements, and velocities, in the case of velocity-dependent devices, shall be determined.

If at least seven ground motions are analyzed, the design values of the damping device forces, displacements, and velocities shall be permitted to be taken as the average of the values determined by the analyses. If fewer than seven ground motions are analyzed, the design damping device forces, displacements, and velocities shall be taken as the maximum value determined by the analysis. A minimum of three ground motions shall be used.

15.3.2 Nonlinear static procedure. The nonlinear modeling described in Sec. A5.2.1 and the lateral loads described in Sec. A5.2.2 shall be applied to the seismic-force-resisting system. The resulting force-displacement curve shall be used in lieu of the assumed effective yield displacement, D_y , of Eq. 15.6-10 to calculate the effective ductility demand due to the design earthquake, μ_D , and due to the maximum considered earthquake, μ_M , in Equations 15.6-8 and 15.6-9, respectively. The value of (R/C_d) shall be taken as 1.0 in Eq. 15.4-4, 15.4-5, 15.4-8, and 15.4-9 for the response spectrum procedure, and in Eq. 15.5-6, 15.5-7 and 15.5-15 for the equivalent lateral force procedure.

15.4 RESPONSE SPECTRUM PROCEDURE

Where the response spectrum procedure is used to design structures with a damping system, the requirements of this section shall apply.

15.4.1 Modeling. A mathematical model of the seismic-force-resisting system and damping system shall be constructed that represents the spatial distribution of mass, stiffness and damping throughout the structure. The model and analysis shall comply with the requirements of Sec. 5.3.1 through 5.3.3 for the seismic-force-resisting system and to the requirements of this section for the damping system. The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 15.9.

The elastic stiffness of elements of the damping system other than damping devices shall be explicitly modeled. Stiffness of damping devices shall be modeled depending on damping device type as follows:

1. *Displacement-Dependent Damping Devices:* Displacement-dependent damping devices shall be modeled with an effective stiffness that represents damping device force at the response displacement of interest (e.g., design story drift). Alternatively, the stiffness of hysteretic and friction damping devices may be excluded from response spectrum analysis provided design forces in displacement-dependent damping devices, Q_{DSD} , are applied to the model as external loads (Sec. 15.7.2.3).
2. *Velocity-Dependent Damping Devices:* Velocity-dependent damping devices that have a stiffness component (e.g., visco-elastic damping devices) shall be modeled with an effective stiffness corresponding to the amplitude and frequency of interest.

15.4.2 Seismic-force-resisting system

15.4.2.1 Seismic base shear. The seismic base shear, V , of the structure in a given direction shall be determined as the combination of modal components, V_m , subject to the limits of Eq. 15.4-1 as follows:

$$V \geq V_{min} \quad (15.4-1)$$

The seismic base shear, V , of the structure shall be determined by the square root sum of the squares or complete quadratic combination of modal base shear components, V_m .

15.4.2.2 Modal base shear. Modal base shear of the m^{th} mode of vibration, V_m , of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-2 as follows:

$$V_m = C_{sm} \bar{W}_m \quad (15.4-2)$$

where:

C_{sm} = seismic response coefficient of the m^{th} mode of vibration of the structure in the direction of interest as determined from Sec. 15.4.2.4 ($m = 1$) or Sec. 15.4.2.6 ($m > 1$), and

\bar{W}_m = the effective gravity load of the m^{th} mode of vibration of the structure determined in accordance with Eq. 5.3-2.

15.4.2.3 Modal participation factor. The modal participation factor of the m^{th} mode of vibration, Γ_m , of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-3 as follows:

$$\Gamma_m = \frac{\bar{W}_m}{\sum_{i=1}^n w_i \phi_{im}} \quad (15.4-3)$$

where:

ϕ_{im} = displacement amplitude at the i^{th} level of the structure for the fixed base condition in the m^{th} mode of vibration in the direction of interest, normalized to unity at the roof level.

15.4.2.4 Fundamental mode seismic response coefficient. The fundamental mode ($m = 1$) seismic response coefficient, C_{s1} , in the direction of interest shall be determined in accordance with Eq. 15.4-4 and 15.4-5 as follows:

$$\text{For } T_{1D} < T_s, C_{s1} = \left(\frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_0 B_{1D}} \quad (15.4-4)$$

$$\text{For } T_{1D} \geq T_s, C_{s1} = \left(\frac{R}{C_d} \right) \frac{S_{D1}}{T_{1D} (\Omega_0 B_{1D})} \quad (15.4-5)$$

15.4.2.5 Effective fundamental mode period determination. The effective fundamental mode ($m = 1$) period at the design earthquake, T_{1D} , and at the maximum considered earthquake, T_{1M} , shall be based either on explicit consideration of the post-yield nonlinear force deflection characteristics of the structure or determined in accordance with Eq. 15.4-6 and 15.4-7 as follows:

$$T_{1D} = T_1 \sqrt{\mu_D} \quad (15.4-6)$$

$$T_{1M} = T_1 \sqrt{\mu_M} \quad (15.4-7)$$

15.4.2.6 Higher mode seismic response coefficient. Higher mode ($m > 1$) seismic response coefficient, C_{Sm} , of the m^{th} mode of vibration ($m > 1$) of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-8 and 15.4-9 as follows:

$$\text{For } T_m < T_s, C_{Sm} = \left(\frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_0 B_{mD}} \quad (15.4-8)$$

$$\text{For } T_m \geq T_s, C_{Sm} = \left(\frac{R}{C_d} \right) \frac{S_{D1}}{T_m (\Omega_0 B_{mD})} \quad (15.4-9)$$

where:

T_m = period, in seconds, of the m^{th} mode of vibration of the structure in the direction under consideration, and

B_{mD} = numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mD} and period of the structure equal to T_m .

15.4.2.7 Design lateral force. Design lateral force at Level i due to m^{th} mode of vibration, F_{im} , of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-10 as follows:

$$F_{im} = w_i \phi_{im} \frac{\Gamma_m}{\bar{W}_m} V_m \quad (15.4-10)$$

Design forces in elements of the seismic-force-resisting system shall be determined by the square root of the sum of the squares or complete quadratic combination of modal design forces.

15.4.3 Damping system. Design forces in damping devices and other elements of the damping system shall be determined on the basis of the floor deflection, story drift and story velocity response parameters described in the following sections.

Displacements and velocities used to determine maximum forces in damping devices at each story shall account for the angle of orientation from horizontal and consider the effects of increased response due to torsion required for design of the seismic-force-resisting system.

Floor deflections at Level i , δ_{iD} and δ_{iM} , design story drifts, Δ_D and Δ_M , and design story velocities, ∇_D and ∇_M , shall be calculated for both the design earthquake and the maximum considered earthquake, respectively, in accordance with this section.

15.4.3.1 Design earthquake floor deflection. The deflection of structure due to the design earthquake at Level i in the m^{th} mode of vibration, δ_{imD} , of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-11 as follows:

$$\delta_{imD} = D_{mD}\phi_{im} \quad (15.4-11)$$

The total design earthquake deflection at each floor of the structure shall be calculated by the square root of the sum of the squares or complete quadratic combination of modal design earthquake deflections.

15.4.3.2 Design earthquake roof displacement. Fundamental ($m = 1$) and higher mode ($m > 1$) roof displacements due to the design earthquake, D_{1D} and D_{mD} , of the structure in the direction of interest shall be determined in accordance with Eq. 15.4-12 and 15.4-13 as follows:

For $m=1$,

$$D_{1D} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DS} T_{1D}^2}{B_{1D}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DS} T_1^2}{B_{1E}}, \quad T_{1D} < T_S \quad (15.4-12a)$$

$$D_{1D} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{D1} T_{1D}}{B_{1D}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{D1} T_1}{B_{1E}}, \quad T_{1D} \geq T_S \quad (15.4-12b)$$

$$\text{For } m > 1, D_{mD} = \left(\frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{D1} T_m}{B_{mD}} \leq \left(\frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{DS} T_m^2}{B_{mD}} \quad (15.4-13)$$

15.4.3.3 Design earthquake story drift. Design earthquake story drift of the fundamental mode, Δ_{1D} , and higher modes, Δ_{mD} ($m > 1$), of the structure in the direction of interest shall be calculated in accordance with Sec. 5.2.6.1 using modal roof displacements of Sec. 15.4.3.2.

Total design earthquake story drift, Δ_D , shall be determined by the square root of the sum of the squares or complete quadratic combination of modal design earthquake drifts.

15.4.3.4 Design earthquake story velocity. Design earthquake story velocity of the fundamental mode, ∇_{1D} , and higher modes, ∇_{mD} ($m > 1$), of the structure in the direction of interest shall be calculated in accordance with Eq. 15.4-14 and 15.4-15 as follows:

$$\text{For } m = 1, \nabla_{1D} = 2\pi \frac{\Delta_{1D}}{T_{1D}} \quad (15.4-14)$$

$$\text{For } m > 1, \nabla_{mD} = 2\pi \frac{A_{mD}}{T_m} \quad (15.4-15)$$

Total design earthquake story velocity, ∇_D , shall be determined by the square root of the sum of the squares or complete quadratic combination of modal design earthquake velocities.

15.4.3.5 Maximum earthquake response. Total modal maximum earthquake floor deflection at Level i , design story drift values and design story velocity values shall be based on Sec. 15.4.3.1, 15.4.3.3 and 15.4.3.4, respectively, except design earthquake roof displacement shall be replaced by maximum earthquake roof displacement. Maximum earthquake roof displacement of the structure in the direction of interest shall be calculated in accordance with Eq. 15.4-16 and 15.4-17 as follows:

For $m=1$,

$$D_{1M} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{MS} T_{1M}^2}{B_{1M}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{MS} T_1^2}{B_{1E}}, \quad T_{1M} < T_S \quad (15.4-16a)$$

$$D_{1M} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{M1} T_{1M}}{B_{1M}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{M1} T_1}{B_{1E}}, \quad T_{1M} \geq T_S \quad (15.4-16b)$$

$$\text{For } m > 1, D_{mM} = \left(\frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{M1} T_m}{B_{mM}} \leq \left(\frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{MS} T_m^2}{B_{mM}} \quad (15.4-17)$$

where:

B_{mM} = numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mM} and period of the structure equal to T_m .

15.5 EQUIVALENT LATERAL FORCE PROCEDURE

Where the equivalent lateral force procedure is used to design structures with a damping system, the requirements of this section shall apply.

15.5.1 Modeling. Elements of the seismic-force-resisting system shall be modeled in a manner consistent with the requirements of Sec. 5.2. For purposes of analysis, the structure shall be considered to be fixed at the base.

Elements of the damping system shall be modeled as required to determine design forces transferred from damping devices to both the ground and the seismic-force-resisting system. The effective stiffness of velocity-dependent damping devices shall be modeled.

Damping devices need not be explicitly modeled provided effective damping is calculated in accordance with the procedures of Sec. 15.6 and used to modify response as required in Sec. 15.5.2 and 15.5.3.

The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 15.9.

15.5.2 Seismic-force-resisting system

15.5.2.1 Seismic base shear. The seismic base shear, V , of the seismic-force-resisting system in a given direction shall be determined as the combination of the two modal components, V_I and V_R , in accordance with the following equation:

$$V = \sqrt{V_I^2 + V_R^2} \geq V_{min} \quad (15.5-1)$$

where:

- V_I = design value of the seismic base shear of the fundamental mode in a given direction of response, as determined in Sec. 15.5.2.2,
- V_R = design value of the seismic base shear of the residual mode in a given direction, as determined in Sec. 15.5.2.6, and
- V_{min} = minimum allowable value of base shear permitted for design of the seismic-force-resisting system of the structure in direction of the interest, as determined in Sec. 15.2.2.1.

15.5.2.2 Fundamental mode base shear. The fundamental mode base shear, V_I , shall be determined in accordance with the following equation:

$$V_I = C_{SI} \bar{W}_I \quad (15.5-2)$$

where:

- C_{SI} = the fundamental mode seismic response coefficient, as determined in Sec. 15.5.2.4, and
- \bar{W}_I = the effective fundamental mode gravity load including portions of the live load as defined by Eq. 5.3-2 for $m = 1$.

15.5.2.3 Fundamental mode properties. The fundamental mode shape, ϕ_{i1} , and participation factor, Γ_I , shall be determined by either dynamic analysis using the elastic structural properties and deformational characteristics of the resisting elements or using Eq. 15.5-3 and 15.5-4 as follows:

$$\phi_{i1} = \frac{h_i}{h_r} \quad (15.5-3)$$

$$\Gamma_I = \frac{\bar{W}_I}{\sum_{i=1}^n w_i \phi_{i1}} \quad (15.5-4)$$

where:

- h_i = the height of the structure above the base to Level i ,
- h_r = the height of the structure above the base to the roof level,
- w_i = the portion of the total gravity load, W , located at or assigned to Level i .

The fundamental period, T_I , shall be determined either by dynamic analysis using the elastic structural properties and deformational characteristics of the resisting elements, or using Eq. 15.5-5 as follows:

$$T_I = 2\pi \sqrt{\frac{\sum_{i=1}^n w_i \delta_i^2}{g \sum_{i=1}^n f_i \delta_i}} \quad (15.5-5)$$

where:

- f_i = lateral force at Level i of the structure distributed in accordance with Sec. 5.2.3, and
- δ_i = elastic deflection at Level i of the structure due to applied lateral forces f_i .

15.5.2.4 Fundamental mode seismic response coefficient. The fundamental mode seismic response coefficient, C_{SI} , shall be determined using Eq. 15.5-6 or 15.5-7 as follows:

$$\text{For } T_{ID} < T_S, C_{SI} = \left(\frac{R}{C_d} \right) \frac{S_{DI}}{\Omega_0 B_{1D}} \quad (15.5-6)$$

$$\text{For } T_{ID} \geq T_S, C_{SI} = \left(\frac{R}{C_d} \right) \frac{S_{DI}}{T_{ID} (\Omega_0 B_{ID})} \quad (15.5-7)$$

where:

- S_{DS} = the design spectral response acceleration parameter in the short period range,
 S_{DI} = the design spectral response acceleration parameter at a period of 1 second, and
 B_{ID} = numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mD} ($m = 1$) and period of the structure equal to T_{ID} .

15.5.2.5 Effective fundamental mode period determination. The effective fundamental mode period at the design earthquake, T_{ID} , and at the maximum considered earthquake, T_{IM} , shall be based on explicit consideration of the post-yield force deflection characteristics of the structure or shall be calculated using Eq. 15.5-8 and 15.5-9 as follows:

$$T_{ID} = T_I \sqrt{\mu_D} \quad (15.5-8)$$

$$T_{IM} = T_I \sqrt{\mu_M} \quad (15.5-9)$$

15.5.2.6 Residual mode base shear. Residual mode base shear, V_R , shall be determined in accordance with Eq. 15.5-10 as follows:

$$V_R = C_{SR} \bar{W}_R \quad (15.5-10)$$

where:

- C_{SR} = the residual mode seismic response coefficient as determined in Sec. 15.5.2.8, and
 \bar{W}_R = the effective residual mode gravity load of the structure determined using Eq. 15.5-13.

15.5.2.7 Residual mode properties. Residual mode shape, ϕ_R , participation factor, Γ_R , effective gravity load of the structure, \bar{W}_R , and effective period, T_R , shall be determined using Eq. 15.5-11 through 15.5-14 as follows:

$$\phi_{iR} = \frac{1 - \Gamma_I \phi_{iI}}{1 - \Gamma_I} \quad (15.5-11)$$

$$\Gamma_R = 1 - \Gamma_I \quad (15.5-12)$$

$$\bar{W}_R = W - \bar{W}_I \quad (15.5-13)$$

$$T_R = 0.4T_I \quad (15.5-14)$$

15.5.2.8 Residual mode seismic response coefficient. The residual mode seismic response coefficient, C_{SR} , shall be determined in accordance with the following equation:

$$C_{SR} = \left(\frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_0 B_R} \quad (15.5-15)$$

where:

- B_R = Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_R , and period of the structure equal to T_R .

15.5.2.9 Design lateral force. The design lateral force in elements of the seismic-force-resisting system at Level i due to fundamental mode response, F_{iI} , and residual mode response, F_{iR} , of the structure in the direction of interest shall be determined in accordance with Eq. 15.5-16 and 15.5-17 as follows:

$$F_{iI} = w_i \phi_{iI} \frac{\Gamma_I}{\bar{W}_I} V_I \quad (15.5-16)$$

$$F_{iR} = w_i \phi_{iR} \frac{\Gamma_R}{\bar{W}_R} V_R \quad (15.5-17)$$

Design forces in elements of the seismic-force-resisting system shall be determined by taking the square root of the sum of the squares of the forces due to fundamental and residual modes.

15.5.3 Damping system. Design forces in damping devices and other elements of the damping system shall be determined on the basis of the floor deflection, story drift, and story velocity response parameters described in the following sections.

Displacements and velocities used to determine maximum forces in damping devices at each story shall account for the angle of orientation from horizontal and consider the effects of increased response due to torsion required for design of the seismic-force-resisting system.

Floor deflections at Level i , δ_{iD} and δ_{iM} , design story drifts, Δ_D and Δ_M , and design story velocities, ∇_D and ∇_M , shall be calculated for both the design earthquake and the maximum considered earthquake, respectively, in accordance with the following sections.

15.5.3.1 Design earthquake floor deflection. The total design earthquake deflection at each floor of the structure in the direction of interest shall be calculated as the square root of the sum of the squares of the fundamental and residual mode floor deflections. The fundamental and residual mode deflections due to the design earthquake, δ_{iID} and δ_{iRD} , at the center of rigidity of Level i of the structure in the direction of interest shall be determined using Eq. 15.5-18 and 15.5-19 as follows:

$$\delta_{iID} = D_{iD} \phi_{iI} \quad (15.5-18)$$

$$\delta_{iRD} = D_{iRD} \phi_{iR} \quad (15.5-19)$$

where:

D_{iD} = Fundamental mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.2.

D_{iRD} = Residual mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.2.

15.5.3.2 Design earthquake roof displacement. Fundamental and residual mode displacements due to the design earthquake, D_{iD} and D_{iR} , at the center of rigidity of the roof level of the structure in the direction of interest shall be determined using Eq. 15.5-20 and 15.5-21 as follows:

$$D_{iD} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DS} T_{iD}^2}{B_{iD}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DS} T_1^2}{B_{1E}}, \quad T_{iD} < T_S \quad (15.5-20a)$$

$$D_{iD} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{D1} T_{iD}}{B_{iD}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{D1} T_1}{B_{1E}}, \quad T_{iD} \geq T_S \quad (15.5-20b)$$

$$D_{iRD} = \left(\frac{g}{4\pi^2} \right) \Gamma_R \frac{S_{D1} T_{iR}}{B_R} \leq \left(\frac{g}{4\pi^2} \right) \Gamma_R \frac{S_{DS} T_R^2}{B_R}$$

15.5.3.3 Design earthquake story drift. Design earthquake story drifts, Δ_D , in the direction of interest shall be calculated using Eq. 15.5-22 as follows:

$$\Delta_D = \sqrt{\Delta_{ID}^2 + \Delta_{RD}^2} \quad (15.5-22)$$

where:

Δ_{ID} = design earthquake story drift due to the fundamental mode of vibration of the structure in the direction of interest, and

Δ_{RD} = design earthquake story drift due to the residual mode of vibration of the structure in the direction of interest.

Modal design earthquake story drifts, Δ_{ID} and Δ_{RD} , shall be determined in accordance with Eq. 5.3-8 using the floor deflections of Sec. 15.5.3.1

15.5.3.4 Design earthquake story velocity. Design earthquake story velocities, ∇_D , in the direction of interest shall be calculated in accordance with Eq. 15.5-23 through 15.5-25 as follows:

$$\nabla_D = \sqrt{\nabla_{ID}^2 + \nabla_{RD}^2} \quad (15.5-23)$$

$$\nabla_{ID} = 2\pi \frac{\Delta_{ID}}{T_{ID}} \quad (15.5-24)$$

$$\nabla_{RD} = 2\pi \frac{\Delta_{RD}}{T_R} \quad (15.5-25)$$

where:

∇_{ID} = design earthquake story velocity due to the fundamental mode of vibration of the structure in the direction of interest, and

∇_{RD} = design earthquake story velocity due to the residual mode of vibration of the structure in the direction of interest.

15.5.3.5 Maximum earthquake response. Total and modal maximum earthquake floor deflections at Level i , design story drifts, and design story velocities shall be based on the equations in Sec. 15.5.3.1, 15.5.3.3 and 15.5.3.4, respectively, except that design earthquake roof displacements shall be replaced by maximum earthquake roof displacements. Maximum earthquake roof displacements shall be calculated in accordance with Eq. 15.5-26 and 15.5-27 as follows:

$$D_{1M} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{MS} T_{1M}^2}{B_{1M}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{MS} T_1^2}{B_{1E}}, \quad T_{1M} < T_S \quad (15.5-26a)$$

$$D_{1M} = \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{M1} T_{1M}}{B_{1M}} \geq \left(\frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{M1} T_1}{B_{1E}}, \quad T_{1M} \geq T_S \quad (15.5-26b)$$

$$D_{RM} = \left(\frac{g}{4\pi^2} \right) \Gamma_R \frac{S_{M1} T_R}{B_R} \leq \left(\frac{g}{4\pi^2} \right) \Gamma_R \frac{S_{MS} T_R^2}{B_R} \quad (15.5-27)$$

where:

- S_{MI} = the maximum considered earthquake, 5-percent-damped, spectral response acceleration at a period of 1 second, determined in accordance with Chapter 3.
- S_{MS} = the maximum considered earthquake, 5-percent-damped, spectral response acceleration at short periods, determined in accordance with Chapter 3.
- B_{IM} = Numerical coefficient as set forth in Table 15.6-1 for effective damping equal to β_{mM} ($m = 1$) and period of structure equal to T_{IM} .

15.6 DAMPED RESPONSE MODIFICATION

As required in Sec. 15.4 and 15.5, response of the structure shall be modified for the effects of the damping system.

15.6.1 Damping coefficient. Where the period of the structure is greater than or equal to T_0 , the damping coefficient shall be as prescribed in Table 15.6-1. Where the period of the structure is less than T_0 , the damping coefficient shall be linearly interpolated between a value of 1.0 at a 0-second period for all values of effective damping and the value at period T_0 as indicated in Table 15.6-1.

Table 15.6-1
Damping Coefficient, B_{V+I} , B_{ID} , B_R , B_{IM} , B_{mD} , or B_{mM}

Effective Damping, β (percentage of critical)	B_{V+I} , B_{ID} , B_R , B_{IM} , B_{mD} or B_{mM} (where period of the structure $\leq T_0$)
≤ 2	0.8
5	1.0
10	1.2
20	1.5
30	1.8
40	2.1
50	2.4
60	2.7
70	3.0
80	3.3
90	3.6
≤ 100	4.0

15.6.2 Effective damping. The effective damping at the design displacement, β_{mD} , and at the maximum displacement, β_{mM} , of the m^{th} mode of vibration of the structure in the direction under consideration shall be calculated using Eq. 15.6-1 and 15.6-2 as follows:

$$\beta_{mD} = \beta_I + \beta_{Vm} \sqrt{\mu_D} + \beta_{HD} \quad (15.6-1)$$

$$\beta_{mM} = \beta_I + \beta_{Vm} \sqrt{\mu_M} + \beta_{HM} \quad (15.6-2)$$

where:

- β_{HD} = component of effective damping of the structure in the direction of interest due to post-yield hysteretic behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand, μ_D ;

- β_{HM} = component of effective damping of the structure in the direction of interest due to post-yield hysteretic behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand, μ_M ;
- β_I = component of effective damping of the structure due to the inherent dissipation of energy by elements of the structure, at or just below the effective yield displacement of the seismic-force-resisting system;
- β_{Vm} = component of effective damping of the m^{th} mode of vibration of the structure in the direction of interest due to viscous dissipation of energy by the damping system, at or just below the effective yield displacement of the seismic-force-resisting system;
- μ_D = effective ductility demand on the seismic-force-resisting system in the direction of interest due to the design earthquake; and
- μ_M = effective ductility demand on the seismic-force-resisting system in the direction of interest due to the maximum considered earthquake.

Unless analysis or test data supports other values, the effective ductility demand of higher modes of vibration in the direction of interest shall be taken as 1.0.

15.6.2.1 Inherent damping. Inherent damping, β_I , shall be based on the material type, configuration, and behavior of the structure and nonstructural components responding dynamically at or just below yield of the seismic-force-resisting system. Unless analysis or test data supports other values, inherent damping shall be taken as not greater than five percent of critical for all modes of vibration.

15.6.2.2 Hysteretic damping. Hysteretic damping of the seismic-force-resisting system and elements of the damping system shall be based either on test or analysis, or shall be calculated using Eq. 15.6-3 and 15.6-4 as follows:

$$\beta_{HD} = q_H (0.64 - \beta_I) \left(1 - \frac{1}{\mu_D} \right) \quad (15.6-3)$$

$$\beta_{HM} = q_H (0.64 - \beta_I) \left(1 - \frac{1}{\mu_M} \right) \quad (15.6-4)$$

where:

- q_H = hysteresis loop adjustment factor, as defined in Sec. 15.6.2.2.1,
- μ_D = effective ductility demand on the seismic-force-resisting system in the direction of interest due to the design earthquake, as defined in Sec. 15.6.3, and
- μ_M = effective ductility demand on the seismic-force-resisting system in the direction of interest due to the maximum considered earthquake, as defined in Sec. 15.6.3.

Unless analysis or test data supports other values, the hysteretic damping of higher modes of vibration in the direction of interest shall be taken as zero.

15.6.2.2.1 Hysteresis loop adjustment factor. The calculation of hysteretic damping of the seismic-force-resisting system and elements of the damping system shall consider pinching and other effects that reduce the area of the hysteresis loop during repeated cycles of earthquake demand. Unless analysis or test data support other values, the fraction of full hysteretic loop area of the seismic-force-resisting system used for design shall be taken as equal to the factor, q_H , using Eq. 15.6-5 as follows:

$$q_H = 0.67 \frac{T_S}{T_I} \quad (15.6-5)$$

where:

- T_S = period defined by the ratio, S_{D1}/S_{DS}

T_1 = period of the fundamental mode of vibration of the structure in the direction of the interest

The value of q_H shall not be taken as greater than 1.0, and need not be taken as less than 0.5.

15.6.2.3 Viscous damping. Viscous damping of the m^{th} mode of vibration of the structure, β_{Vm} , shall be calculated using Eq. 15.6-6 and 15.6-7 as follows:

$$\beta_{Vm} = \frac{\sum_j W_{mj}}{4\pi W_m} \quad (15.6-6)$$

$$W_m = \frac{1}{2} \sum_i F_{im} \delta_{im} \quad (15.6-7)$$

where:

W_{mj} = work done by j^{th} damping device in one complete cycle of dynamic response corresponding to the m^{th} mode of vibration of the structure in the direction of interest at modal displacements, δ_{im} ,

W_m = maximum strain energy in the m^{th} mode of vibration of the structure in the direction of interest at modal displacements, δ_{im} ,

F_{im} = m^{th} mode inertial force at Level i ,

δ_{im} = deflection of Level i in the m^{th} mode of vibration at the center of rigidity of the structure in the direction under consideration.

Viscous modal damping of displacement-dependent damping devices shall be based on a response amplitude equal to the effective yield displacement of the structure.

The calculation of the work done by individual damping devices shall consider orientation and participation of each device with respect to the mode of vibration of interest. The work done by individual damping devices shall be reduced as required to account for the flexibility of elements, including pins, bolts, gusset plates, brace extensions, and other components that connect damping devices to other elements of the structure.

15.6.3 Effective ductility demand. The effective ductility demand on the seismic-force-resisting system due to the design earthquake, μ_D , and due to the maximum considered earthquake, μ_M , shall be calculated using Eq. 15.6-8, 15.6-9, and 15.6-10 as follows:

$$\mu_D = \frac{D_{1D}}{D_Y} \geq 1.0 \quad (15.6-8)$$

$$\mu_M = \frac{D_{1M}}{D_Y} \geq 1.0 \quad (15.6-9)$$

$$D_Y = \left(\frac{g}{4\pi^2} \right) \left(\frac{\Omega_0 C_d}{R} \right) \Gamma_1 C_{S1} T_1^2 \quad (15.6-10)$$

where:

D_{1D} = fundamental mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 15.5.3.2,

D_{1M} = fundamental mode maximum displacement at the center of rigidity of the roof level of structure in the direction under consideration, Sec. 15.5.3.5,

D_Y = displacement at the center of rigidity of the roof level of the structure at the effective yield point of the seismic-force-resisting system,

R = response modification factor from Table 4.3-1,

- C_d = deflection amplification factor from Table 4.3-1,
 Ω_0 = system overstrength factor from Table 4.3-1,
 Γ_1 = participation factor of the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.4.2.3 or Sec. 15.5.2.3 ($m = 1$),
 C_{SI} = seismic response coefficient of the fundamental mode of vibration of the structure in the direction of interest, Sec. 15.4.2.4 or Sec. 15.5.2.4 ($m = 1$), and
 T_1 = period of the fundamental mode of vibration of the structure in the direction of interest.

The design earthquake ductility demand, μ_D , shall not exceed the maximum value of effective ductility demand, μ_{max} , given in Sec. 15.6.4.

15.6.4 Maximum effective ductility demand. For determination of the hysteresis loop adjustment factor, hysteretic damping, and other parameters, the maximum value of effective ductility demand, μ_{max} , shall be calculated using Eq. 15.6-11 and 15.6-12 as follows:

$$\text{For } T_{1D} \leq T_S, \mu_{max} = \frac{1}{2} \left(\left(\frac{R}{\Omega_0 I} \right)^2 + 1 \right) \quad (15.6-11)$$

$$\text{For } T_1 \geq T_S, \mu_{max} = \frac{R}{\Omega_0 I} \quad (15.6-12)$$

For $T_1 < T_S < T_{1D}$, μ_{max} shall be determined by linear interpolation between the values of Eq. 15.6-11 and 15.6-12

where:

- I = the occupancy importance factor determined in accordance with Sec. 1.3.
 T_{1D} = effective period of the fundamental mode of vibration of the structure at the design displacement in the direction under consideration.

15.7 SEISMIC LOAD CONDITIONS AND ACCEPTANCE CRITERIA

For the nonlinear procedures of Sec. 15.3, the seismic-force-resisting system, damping system, loading conditions and acceptance criteria for response parameters of interest shall conform with Sec. 15.7.1. Design forces and displacements determined in accordance with the response spectrum procedure of Sec. 15.4 or the equivalent lateral force procedure of Sec. 15.5 shall be checked using the strength design criteria of the *Provisions* and the seismic loading conditions of Sec. 15.7.2 and 15.7.3.

15.7.1 Nonlinear procedures. Where nonlinear procedures are used in analysis, the seismic force resisting system, damping system, seismic loading conditions and acceptance criteria shall conform to the following subsections.

15.7.1.1 Seismic force resisting system. The seismic-force-resisting system shall satisfy the strength requirements of Sec. 4.3 using the seismic base shear, V_{min} , as given by Sec. 15.2.2.1.

15.7.1.2 Seismic loads. Seismic forces shall be based upon the design earthquake for damping system strength. The damping devices and their connections shall be sized to resist the forces, displacements and velocities from the maximum considered earthquake. The story drift shall be determined using the design earthquake.

15.7.1.3 Combination of load effects. The effects on the damping system due to gravity loads and seismic forces shall be combined in accordance with Sec. 4.2.2 using the effect of horizontal seismic forces, Q_E , determined in accordance with the analysis. The redundancy factor, ρ , shall be taken equal to 1.0 in all cases and the seismic load effect with overstrength of Sec. 4.2.2.2 need not apply to the design of the damping system.

15.7.1.4 Acceptance criteria for the response parameters of interest. The damping system components shall be evaluated using the strength design criteria of the *Provisions* using the seismic forces and seismic loading conditions determined from the nonlinear procedures and $\phi = 1.0$. The members of the seismic-force-resisting system need not be evaluated when using the nonlinear procedure forces.

The story drift shall not exceed 125 percent of the allowable story drift, Δ_a , as obtained from Table 4.5-1. The maximum story drift shall include torsional effects.

15.7.2 Seismic-force-resisting system. The seismic-force-resisting system shall satisfy the requirements of Sec. 4.3 using seismic base shear and design forces determined in accordance with Sec. 15.4.2 or Sec. 15.5.2.

The design earthquake story drift, Δ_D , as determined in either Sec. 15.4.3.3 or Sec. 15.5.3.3 shall not exceed (R/C_d) times the allowable story drift, as obtained from Table 4.5-1, considering the effects of torsion as required in Sec. 4.5.1.

15.7.3 Damping system. The damping system shall satisfy the requirements of Sec. 4.3 for seismic design forces and seismic loading conditions determined in accordance with this section.

15.7.3.1 Combination of load effects. The effects on the damping system and its components due to gravity loads and seismic forces shall be combined in accordance with Sec. 4.2.2 using the effect of horizontal seismic forces, Q_E , determined in accordance with Sec. 15.7.3.3. The redundancy factor, ρ , shall be taken equal to 1.0 in all cases and the seismic load effect with overstrength of Sec. 4.2.2.2 need not apply to the design of the damping system.

15.7.3.2 Modal damping system design forces. Modal damping system design forces shall be calculated on the basis of the type of damping devices and the modal design story displacements and velocities determined in accordance with either Sec. 15.4.3 or Sec. 15.5.3.

Modal design story displacements and velocities shall be increased as required to envelop the total design story displacements and velocities determined in accordance with Sec. 15.3 where peak response is required to be confirmed by time history analysis.

1. Displacement-Dependent Damping Devices: Design seismic force in displacement-dependent damping devices shall be based on the maximum force in the device at displacements up to and including the design earthquake story drift, Δ_D .
2. Velocity-Dependent Damping Devices: Design seismic force in each mode of vibration of velocity-dependent damping devices shall be based on the maximum force in the device at velocities up to and including the design earthquake story velocity for the mode of interest.

Displacements and velocities used to determine design forces in damping devices at each story shall account for the angle of orientation from horizontal and consider the effects of increased floor response due to torsional motions.

15.7.3.3 Seismic load conditions and combination of modal responses. Seismic design force, Q_E , in each element of the damping system due to horizontal earthquake load shall be taken as the maximum force of the following three loading conditions:

1. Stage of Maximum Displacement: Seismic design force at the stage of maximum displacement shall be calculated in accordance with Eq. 15.7-1 as follows:

$$Q_E = \Omega_o \sqrt{\sum_m (Q_{mSFERS})^2} \pm Q_{DSD} \quad (15.7-1)$$

where:

$$Q_{mSFERS} = \text{Force in an element of the damping system equal to the design seismic force of the } m^{\text{th}} \text{ mode of vibration of the seismic-force-resisting system in the direction of interest.}$$

Q_{DSD} = Force in an element of the damping system required to resist design seismic forces of displacement-dependent damping devices.

Seismic forces in elements of the damping system, Q_{DSD} , shall be calculated by imposing design forces of displacement-dependent damping devices on the damping system as pseudo-static forces. Design seismic forces of displacement-dependent damping devices shall be applied in both positive and negative directions at peak displacement of the structure.

2. Stage of Maximum Velocity: Seismic design force at the stage of maximum velocity shall be calculated in accordance with Eq. 15.7-2 as follows:

$$Q_E = \sqrt{\sum_m (Q_{mDSV})^2} \quad (15.7-2)$$

where:

Q_{mDSV} = Force in an element of the damping system required to resist design seismic forces of velocity-dependent damping devices due to the m^{th} mode of vibration of structure in the direction of interest.

Modal seismic design forces in elements of the damping system, Q_{mDSV} , shall be calculated by imposing modal design forces of velocity-dependent devices on the non-deformed damping system as pseudo-static forces. Modal seismic design forces shall be applied in directions consistent with the deformed shape of the mode of interest. Horizontal restraint forces shall be applied at each floor Level i of the non-deformed damping system concurrent with the design forces in velocity-dependent damping devices such that the horizontal displacement at each level of the structure is zero. At each floor Level i , restraint forces shall be proportional to and applied at the location of each mass point.

3. Stage of Maximum Acceleration: Seismic design force at the stage of maximum acceleration shall be calculated in accordance Eq. 15.7-3 as follows:

$$Q_E = \sqrt{\sum_m (C_{mFD} \Omega_o Q_{mSFRS} + C_{mFV} Q_{mDSV})^2} \pm Q_{DSD} \quad (15.7-3)$$

The force coefficients, C_{mFD} and C_{mFV} , shall be determined from Tables 15.7-1 and 15.7-2, respectively, using values of effective damping determined in accordance with the following requirements:

For fundamental-mode response ($m = 1$) in the direction of interest, the coefficients, C_{1FD} and C_{1FV} , shall be based on the velocity exponent, α , that relates device force to damping device velocity. The effective fundamental-mode damping, shall be taken equal to the total effective damping of the fundamental mode less the hysteretic component of damping ($\beta_{1D} - \beta_{HD}$ or $\beta_{1M} - \beta_{HM}$) at the response level of interest ($\mu = \mu_D$ or $\mu = \mu_M$).

For higher-mode ($m > 1$) or residual-mode response in the direction of interest, the coefficients, C_{mFD} and C_{mFV} , shall be based on a value of α equal to 1.0. The effective modal damping shall be taken equal to the total effective damping of the mode of interest (β_{mD} or β_{mM}). For determination of the coefficient C_{mFD} , the ductility demand shall be taken equal to that of the fundamental mode ($\mu = \mu_D$ or $\mu = \mu_M$).

Table 15.7-1 Force Coefficient, $C_{mFD}^{a,b}$

Effective Damping	$\mu \leq 1.0$				$C_{mFD} = 1.0^c$
	$\alpha \leq 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha \geq 1.0$	
≤ 0.05	1.00	1.00	1.00	1.00	$\mu \geq 1.0$
0.1	1.00	1.00	1.00	1.00	$\mu \geq 1.0$
0.2	1.00	0.95	0.94	0.93	$\mu \geq 1.1$
0.3	1.00	0.92	0.88	0.86	$\mu \geq 1.2$
0.4	1.00	0.88	0.81	0.78	$\mu \geq 1.3$
0.5	1.00	0.84	0.73	0.71	$\mu \geq 1.4$
0.6	1.00	0.79	0.64	0.64	$\mu \geq 1.6$
0.7	1.00	0.75	0.55	0.58	$\mu \geq 1.7$
0.8	1.00	0.70	0.50	0.53	$\mu \geq 1.9$
0.9	1.00	0.66	0.50	0.50	$\mu \geq 2.1$
≥ 1.0	1.00	0.62	0.50	0.50	$\mu \geq 2.2$

^a Unless analysis or test data support other values, the force coefficient C_{mFD} for visco-elastic systems shall be taken as 1.0.

^b Interpolation shall be used for intermediate values of velocity exponent α , and ductility demand, μ .

^c C_{mFD} shall be taken equal to 1.0 for values of ductility demand, μ , greater than or equal to the values shown.

Table 15.7-2 Force Coefficient, $C_{mFV}^{a,b}$

Effective Damping	$\alpha \leq 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha \geq 1.0$
≤ 0.05	1.00	0.35	0.20	0.10
0.1	1.00	0.44	0.31	0.20
0.2	1.00	0.56	0.46	0.37
0.3	1.00	0.64	0.58	0.51
0.4	1.00	0.70	0.69	0.62
0.5	1.00	0.75	0.77	0.71
0.6	1.00	0.80	0.84	0.77
0.7	1.00	0.83	0.90	0.81
0.8	1.00	0.90	0.94	0.90
0.9	1.00	1.00	1.00	1.00
≥ 1.0	1.00	1.00	1.00	1.00

^a Unless analysis or test data support other values, the force coefficient C_{mFD} for visco-elastic systems shall be taken as 1.0.

^b Interpolation shall be used for intermediate values of velocity exponent, α .

15.7.3.4 Inelastic response limits. Elements of the damping system may exceed strength limits for design loads provided it is shown by analysis or test that:

1. Inelastic response does not adversely affect damping system function.
2. Element forces calculated in accordance with Sec. 15.7.3.3, using a value of Ω_0 , taken equal to 1.0, do not exceed the strength required to satisfy the load combinations of Sec. 4.2.2.1.

15.8 DESIGN REVIEW

A design review of the damping system and related test programs shall be performed by an independent team of registered design professionals in the appropriate disciplines and others experienced in seismic analysis methods and the theory of energy dissipation systems.

The design review shall include, but need not be limited to, the following:

1. Review of site-specific seismic criteria including the development of the site-specific spectra and ground motion histories and all other design criteria developed specifically for the project;
2. Review of the preliminary design of the seismic-force-resisting system and the damping system, including design parameters of damping devices;
3. Review of the final design of the seismic-force-resisting system and the damping system and all supporting analyses; and
4. Review of damping device test requirements, device manufacturing quality control and assurance, and scheduled maintenance and inspection requirements.

15.9 TESTING

The force-velocity-displacement and damping properties used for the design of the damping system shall be based on the prototype tests as specified in this section.

The fabrication and quality control procedures used for all prototype and production damping devices shall be identical.

15.9.1 Prototype tests

The following tests shall be performed separately on two full-size damping devices of each type and size used in the design, in the order listed below.

Representative sizes of each type of device may be used for prototype testing, provided both of the following conditions are met:

1. All pertinent testing and other damping device data are made available to, and are accepted by the registered design professional responsible for the design of the structure.
2. The registered design professional substantiates the similarity of the damping device to previously tested devices.

Test specimens shall not be used for construction, unless they are accepted by the registered design professional responsible for design of the structure and meet the requirements for prototype and production tests.

15.9.1.1 Data recording. The force-deflection relationship for each cycle of each test shall be recorded.

15.9.1.2 Sequence and cycles of testing. For the following test sequences, each damping device shall be subjected to gravity load effects and thermal environments representative of the installed condition. For seismic testing, the displacement in the devices calculated for the maximum considered earthquake, termed herein as the maximum earthquake device displacement, shall be used.

1. Each damping device shall be subjected to the number of cycles expected in the design windstorm, but not less than 2000 continuous fully reversed cycles of wind load. Wind load shall be at

amplitudes expected in the design wind storm, and applied at a frequency equal to the inverse of the fundamental period of the building ($f_l = 1/T_l$).

Exception: Damping devices need not be subjected to these tests if they are not subject to wind-induced forces or displacements, or if the design wind force is less than the device yield or slip force.

2. Each damping device shall be loaded with 5 fully reversed, sinusoidal cycles at the maximum earthquake device displacement at a frequency equal to $1/T_{IM}$ as calculated in Sec. 15.4.2.5. Where the damping device characteristics vary with operating temperature, these tests shall be conducted at a minimum of 3 temperatures (minimum, ambient, and maximum) that bracket the range of operating temperatures.

Exception: Damping devices may be tested by alternative methods provided all of the following conditions are met:

- a. Alternative methods of testing are equivalent to the cyclic testing requirements of this section.
 - b. Alternative methods capture the dependence of the damping device response on ambient temperature, frequency of loading, and temperature rise during testing.
 - c. Alternative methods are accepted by the registered design professional responsible for the design of the structure.
3. If the force-deformation properties of the damping device at any displacement less than or equal the maximum earthquake device displacement change by more than 15 percent for changes in testing frequency from $1/T_{IM}$ to $2.5/T_l$, then the preceding tests shall also be performed at frequencies equal to $1/T_l$ and $2.5/T_l$.

If reduced-scale prototypes are used to qualify the rate dependent properties of damping devices, the reduced-scale prototypes should be of the same type and materials, and manufactured with the same processes and quality control procedures, as full-scale prototypes, and tested at a similitude-scaled frequency that represents the full-scale loading rates.

15.9.1.3 Testing similar devices. Damping devices need not be prototype tested provided that both of the following conditions are met:

1. All pertinent testing and other damping device data are made available to, and are accepted by the registered design professional responsible for the design of the structure.
2. The registered design professional substantiates the similarity of the damping device to previously tested devices.

15.9.1.4 Determination of force-velocity-displacement characteristics. The force-velocity-displacement characteristics of a damping device shall be based on the cyclic load and displacement tests of prototype devices specified above. Effective stiffness of a damping device shall be calculated for each cycle of deformation using equation 13.6-1.

15.9.1.5 Device adequacy. The performance of a prototype damping device shall be deemed adequate if all of the conditions listed below are satisfied. The 15-percent limits specified below may be increased by the registered design professional responsible for the design of the structure provided that the increased limit has been demonstrated by analysis not to have a deleterious effect on the response of the structure.

15.9.1.5.1 Displacement-dependent damping devices. The performance of the prototype displacement-dependent damping devices shall be deemed adequate if the following conditions, based on tests specified in Sec. 15.9.1.2, are satisfied:

1. For Test 1, no signs of damage including leakage, yielding, or breakage.
2. For Tests 2 and 3, the maximum force and minimum force at zero displacement for a damping device for any one cycle does not differ by more than 15 percent from the average maximum and minimum forces at zero displacement as calculated from all cycles in that test at a specific frequency and temperature.
3. For Tests 2 and 3, the maximum force and minimum force at maximum earthquake device displacement for a damping device for any one cycle does not differ by more than 15 percent from the average maximum and minimum forces at the maximum earthquake device displacement as calculated from all cycles in that test at a specific frequency and temperature.
4. For Tests 2 and 3, the area of hysteresis loop (E_{loop}) of a damping device for any one cycle does not differ by more than 15 percent from the average area of the hysteresis loop as calculated from all cycles in that test at a specific frequency and temperature.
5. The average maximum and minimum forces at zero displacement and maximum earthquake displacement, and the average area of the hysteresis loop (E_{loop}), calculated for each test in the sequence of Tests 2 and 3, shall not differ by more than 15 percent from the target values specified by the registered design professional responsible for the design of the structure.

15.9.1.5.1 Velocity-dependent damping devices. The performance of the prototype velocity-dependent damping devices shall be deemed adequate if the following conditions, based on tests specified in Sec. 15.9.1.2, are satisfied:

1. For Test 1, no signs of damage including leakage, yielding, or breakage.
2. For velocity-dependent damping devices with stiffness, the effective stiffness of a damping device in any one cycle of Tests 2 and 3 does not differ by more than 15 percent from the average effective stiffness as calculated from all cycles in that test at a specific frequency and temperature.
3. For Tests 2 and 3, the maximum force and minimum force at zero displacement for a damping device for any one cycle does not differ by more than 15 percent from the average maximum and minimum forces at zero displacement as calculated from all cycles in that test at a specific frequency and temperature.
4. For Tests 2 and 3, the area of hysteresis loop (E_{loop}) of a damping device for any one cycle does not differ by more than 15 percent from the average area of the hysteresis loop as calculated from all cycles in that test at a specific frequency and temperature.
5. The average maximum and minimum forces at zero displacement, effective stiffness (for damping devices with stiffness only), and average area of the hysteresis loop (E_{loop}) calculated for each test in the sequence of Tests 2 and 3, does not differ by more than 15 percent from the target values specified by the registered design professional responsible for the design of the structure.

15.9.2 Production testing. Prior to installation in a building, damping devices shall be tested to ensure that their force-velocity-displacement characteristics fall within the limits set by the registered design professional responsible for the design of the structure. The scope and frequency of the production-testing program shall be determined by the registered design professional responsible for the design of the structure.