Appendix to Chapter 5

NONLINEAR STATIC PROCEDURE

A5.2 NONLINEAR STATIC PROCEDURE

The nonlinear static procedure is intended to provide a simplified approach for directly determining the nonlinear response behavior of a structure at different levels of lateral displacements, ranging from initial elastic response through development of a failure mechanism and initiation of collapse. Response behavior is gauged by measurement of the strength of the structure, at various increments of lateral displacement.

Usually the shear resisted by the system at yield of the first element of the structure is defined as the "elastic strength," although this may not correspond to yield of the entire structure. When traditional linear methods of design (using *R* factors) are employed, this elastic strength will not be less than the design base shear.

If a structure is subjected to larger lateral loads than that represented by the elastic strength, a number of elements will yield—eventually forming a mechanism. For most structures, multiple configurations of mechanisms are possible. The mechanism caused by the smallest set of forces is likely to appear before others do. That mechanism is considered to be the dominant mechanism. Standard methods of plastic or "limit" analysis can be used to determine the strength corresponding to such mechanisms. However, such "limit analysis" cannot determine the deformation at the onset of such a mechanism. If the yielding elements are able to strain harden, the mechanism will not allow an increase of deformations without some increase of lateral forces and the mechanism is stable. Moreover, it can be considered as a flexible version of the original frame structure. Figure CA5.2-1, which shows a plot of the lateral structural strength vs. deformation (or pushover curve) for a hypothetical structure, illustrates these concepts.

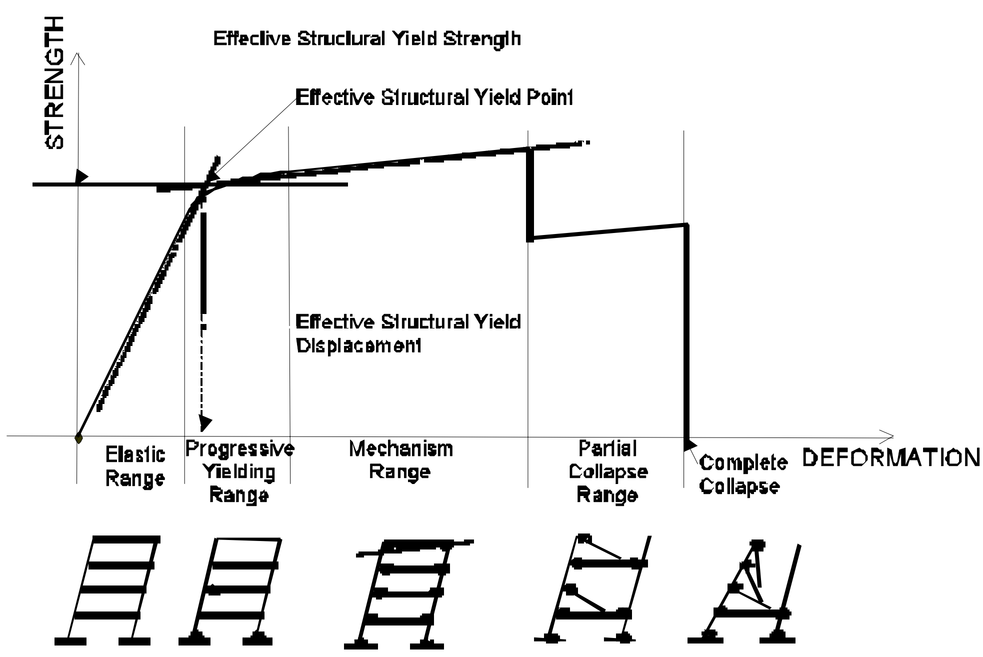

Figure CA5.2-1 Strength-deformation relation for a frame structure

If, after the structure develops a mechanism, it deforms an additional substantial amount, elements within the structure may fail (fracture, buckle, etc.) and thus cease to contribute strength to the structural system. In such cases, the strength of the structure will diminish with increasing deformation. In the event of failure of an essential element, or group of elements, the entire structure may lose capacity to carry the gravity or lateral loads. Such loss of load-carrying capacity can also occur if the lateral deformation becomes so great that the *P*-delta effects exceed the residual lateral stiffness of the structure. Such conditions are defined as collapse and the deformation associated with collapse is defined as the "ultimate deformation." This deformation can be determined by the nonlinear static procedure and also by plastic or limit analysis.

As shown in Figure CA5.2-1, many structures exhibit a range of behavior between the development of first yielding and development of a mechanism. When the structure deforms while elements are yielding sequentially (shown as progressive yielding), the relation between external forces and deformations cannot be determined by simple limit analysis. For such a case, other methods of analysis are required. The purpose of nonlinear static procedure is to provide a simplified method of determining structural response behavior at deformation levels between those that can be conveniently analyzed using limit state methods.

A5.2.1 Modeling. In this procedure, the structure is modeled using elements having stiffness properties that are dependent on the amount of deformation imposed on the element. All elements than can be subjected to deformations or forces larger than those corresponding to yield should be modeled with nonlinear properties. At a minimum, nonlinear stiffness properties (using a bilinear model) should include initial elastic stiffness, yield strength (and yield deformation), and post-yield characteristics including the point of loss of strength (and associated deformation) or point of complete fracture or loss of stability.

A5.2.2 Lateral loads. The analysis is performed by applying a incrementally increasing pattern of lateral loads distributed throughout the structure. The analysis traces the internal distribution of loads and deformations as the load amplitude is progressively increased. Moreover it records the strength-deformation relation and the characteristic events occurring as the analysis progresses. The strength-deformation relation typically takes a shape similar to that shown in Figure CA5.2-1.

It should be noted that nonlinear static analysis can be used to determine the order of yielding of elements in the "progressive yielding range" (see Figure CA5.2-1) and the associated strengths and deformations. The analysis can also identify the deformations associated with fractures or failure of components and the entire structure. However, it is accurate only if the applied pattern of loads induces a pattern of deformation in the structure that is similar to that which will be induced by the earthquake ground motion. This can be controlled, to some extent, through application of an appropriate pattern of loads. However, this method is generally limited in applicability to structures that have limited higher-mode participation.

The force-deformation sequence predicted by the analysis is a function of the configuration of the set of monotonically increasing loads. In order to capture the dynamic behavior of the structure, the force-deformation relation should be properly defined as the instantaneous distribution of inertial forces when the maximum response of structure occurs. Therefore, the load configuration should be redefined at each point on the pushover curve, proportional to the instantaneous configuration of inertial forces. Such a configuration is dependent on the instantaneous modal characteristics of the structure and their combination. Since the structure is nonlinear, the instantaneous modal characteristics depend on the modified properties due to inelastic deformations, affecting the load distribution at each step, accordingly.

Such use of a varying, deformation-dependent load configuration would require almost as much labor and uncertainty as application of a full nonlinear response history procedure. Such effort would be inappropriate for the simplified approach that the nonlinear static procedure is intended to provide. Therefore, the load configuration and intensity are approximated in the nonlinear static procedures. Several approximations are available, including the following:

1. An approximate distribution proportional to the idealized elastic response model as used in the equivalent lateral force procedure:

$$F_i = \frac{w_i h_i^k}{\sum_j w_j h_i^k} V \qquad \text{(CA5.2-1)}$$

where, F, w, h and V are the story inertia force, story weight, story height, and base shear, respectively; k is a coefficient ranging between 1 and 2, as defined in *Provisions* Sec. 5.2.3.

2. A better approximation, using the dominant mode of vibration (such as the first mode in moderate height building structures):

$$F_i = \frac{w_i \phi_i}{\sum_i w_i \phi_i} V \qquad \text{(CA5.2-2)}$$

where, ϕ_i is the dominant mode shape. This approximation allows the three-dimensional distribution of inertia forces to be obtained when such considerations are important.

3. A still more complete approximation using several significant modes of vibration. In such cases the modes for which the total equivalent modal mass exceed 90 percent should be included. The load configuration is given by:

$$F_i = \frac{w_i \phi_{id}}{\Sigma w_i \phi_{id}} \frac{\left[\Sigma\left[\left(\Gamma_i/\Gamma_d\right)\left(S_{ai}/S_{ad}\right)\right]^2\right]^{1/2}}{\left[\Sigma\left[\left(\Gamma_i/\Gamma_d\right)^2\left(S_{ai}/S_{ad}\right)\right]^2\right]^{1/2}} V \qquad \text{(CA5.2-3)}$$

where, Γ_i and S_{ai} are the modal participation factor and the spectral acceleration, respectively, and subscript d indicates the dominant mode. ($\Gamma_i = \Sigma w_i \phi_i$; where the mode shapes, ϕ, are mass normalized—that is $\sum w_i \phi_i^2 / g = 1$.)

4. An approximation that takes into account both higher mode contributions and changes in the loading due to yielding of the structure. In this case the load configuration described by Eq. CA5.2-3) is calculated and reevaluated when the modal characteristics of the structure change as it yields. Such procedure has also termed an "adaptive push-over analysis."

The *Provisions* adopt the simplest of these approaches, indicated as item 1 above, though use of the more complex approaches is not precluded. Nonlinear static analysis options exist in several commercially available and public-domain analysis platforms.

A5.2.3 Target displacement. The nonlinear analysis should be continued by increasing the amplitude of the pattern of lateral loads until the deflections at the control point exceeds 150 percent of the target displacement. The expected inelastic deflection at each level shall be determined by combining the elastic modal values as obtained from Sec. 5.3.5 and 5.3.6 multiplied by the factor

$$C = \frac{\left(1 - T_s/T_1\right)}{R_d} + \left(T_s/T_1\right) \qquad \text{(CA5.2-4)}$$

where T_s is the characteristic period of the response spectrum, defined as the period associated with the transition from the constant-acceleration segment of the spectrum to the constant-velocity segment of the spectrum and R_d is the ratio of the total design base shear to the fully yielded strength of the major mechanism, which can be obtained according to $R_d = R/\Omega_o$, with R and Ω_o given in Table 4.3-1. The combination shall be carried out by taking the square root of the sum of the squares of each of the modal values or by the complete quadratic combination technique.

The recommendation linking the expected inelastic deformation to the elastic is based on an approach originally suggested by Newmark and on later studies by several other researchers. These are described below.

In a 1991 study, Nassar and Krawinkler published simplified expressions that were derived from a study of mean strength reduction factors computed from fifteen ground motions recorded in the Western United States. The records used were obtained at alluvium and rock sites. The influence of the site conditions was not explicitly considered. The sensitivity of mean strength reduction factors to epicentral distance, yield level, strain-hardening ratio, and stiffness degradation was examined. The study concluded that epicentral distance and stiffness degradation have negligible influence on strength reduction factors and proposed the following relationship for the ratio of inelastic displacements to displacements predicted by elastic analysis:

$$R_d = \left[1 + \frac{1}{c}\left(r^c - 1\right)\right] / r \geq 1 \qquad \text{(CA5.2-5)}$$

where,

$$c = \frac{T^a}{1 + T^a} + \frac{b}{T} \qquad \text{(CA5.2-6)}$$

In the above, T, is the period of vibration of the structure and r is the strength ratio. R_d is defined above.

In 1994, Chang and Mander performed analytical studies based on an envelope of five recorded ground motions. The following inelastic dynamic magnification factor that relates the maximum inelastic displacement to the elastic spectral displacement was obtained.

$$R_D = \left(1 - \frac{1}{r}\right)\left(\frac{T_{PV}}{T}\right)^n + \frac{1}{r} \cdot \geq 1 \qquad \text{(CA5.2-7)}$$

where T_{PV} is the period at which the maximum spectral velocity response occurs, and

$n = 1.2 + 0.025r$ for $T_{PV} \leq 1.2$ sec. (CA5.2-8)

$n = 1.2$ for $T_{PV} > 1.2$ sec. (CA5.2-9)

In 1992, Vidic, Fajfar, and Fischinger recommended simplified expressions derived from the study of the mean strength reduction factors computed from twenty ground motions recorded in the Western United States as well as in the 1979 Montenegro, Yugoslavia, earthquake. Systems with bilinear and stiffness degrading (Q-model) hysteric behavior and viscous damping proportional to the mass and the instantaneous stiffness were considered, resulting in the following expression:

$$R_D = \left(1 - \frac{1}{r}\right)\frac{T_0}{T} + \frac{1}{r} \geq 1 \qquad \text{(CA5.2-10)}$$

where T is the dominant period of structure, $T_0 = 0.65\mu^{0.3}T_1$, and

$$T_1 = 2\pi \frac{\phi_{ev}}{\phi_{ea}} \frac{V}{A} \qquad \text{(CA5.2-11)}$$

where V and A are the peak ground velocity and peak ground acceleration, respectively. For the 20 ground motions considered in the study, the mean amplification factors ϕ_{ea} and ϕ_{ev} are 2.5 and 2.0, respectively.

Miranda and Bertero (1994) suggested simplified expressions derived from the study of the mean strength reduction factors computed from 124 ground motions recorded on a wide range of soil conditions. The study considered 5-percent-damped bilinear systems undergoing displacement ductility ratios between 2 and 6. Based on the local site conditions at the recording station, ground motions were classified into three groups: rock sites, and soft soil sites. In addition to the influence of soil conditions, the study considered the influence of magnitude and epicentral distance on strength reduction factors. The study concluded that soil conditions influence the reduction factors significantly (particularly for soft soil sites) and that magnitude and epicentral distance have a negligible effect on mean strength reduction factors. The study produced the following expression for the mean strength reduction factor:

$$R_D = \left(1 - \frac{1}{r}\right)\Phi + \frac{1}{r} \tag{CA5.2-12}$$

with,

$$\Phi = 1 + \frac{1}{10T - \mu T} - \frac{1}{2T}\exp\left[-\frac{3}{2}\left(\ln T - \frac{3}{5}\right)^2\right] \tag{CA5.2-13}$$

$$\Phi = 1 + \frac{1}{12T - \mu T} - \frac{2}{5T}\exp\left[-2\left(\ln T - \frac{1}{5}\right)^2\right] \tag{CA5.2-14}$$

(CA5.2-15)

where T is the period of vibration of the structure and T_g is the characteristic ground motion period.

The recommended formulation contained in the *Provisions* is a combination of the recommendations of Krawinkler et al and of Vidic et al with some simplification. The *Provisions* require that the analysis be continued until the deflection at the control point exceeds 150 percent of the target displacement in order to account for inaccuracy due to this simplification and because small variations in strength (due to modeling or due to imprecise construction) can lead to large displacement variations in the inelastic range.

A5.2.5 Design review. See *Commentary* Sec. 5.5.4.

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