Chapter 5 Commentary

STRUCTURAL ANALYSIS PROCEDURES

5.1 GENERAL

The equivalent lateral force (ELF) procedure specified in Sec. 5.2 is similar in its basic concept to SEAOC recommendations in 1968, 1973, and 1974, but several improved features have been incorporated. A significant revision to this procedure, which more closely reflects the ground motion response spectra, was adopted in the 1997 *Provisions* in parallel with a similar concept developed by SEAOC.

The modal superposition method is a general procedure for linear analysis of the dynamic response of structures. In various forms, modal analysis has been widely used in the earthquake-resistant design of special structures such as very tall buildings, offshore drilling platforms, dams, and nuclear power plants, for a number of years; however, its use is also becoming more common for ordinary structures as well. Prior to the 1997 edition of the *Provisions*, the modal analysis procedure specified in Sec. 5.3 was simplified from the general case by restricting consideration to lateral motion in a single plane. Only one degree of freedom was required per floor for this type of analysis. In recent years, with the advent of high-speed, desktop computers, and the proliferation of relatively inexpensive, user-friendly structural analysis software capable of performing three dimensional modal analyses, such simplifications have become unnecessary. Consequently, the 1997 *Provisions* adopted the more general approach describing a three-dimensional modal analysis of the structure. When modal analysis is specified by the *Provisions*, a three-dimensional analysis generally is required except in the case of highly regular structures or structures with flexible diaphragms.

The ELF procedure of Sec. 5.2 and the response spectrum procedure of Sec. 5.3 are both based on the approximation that the effects of yielding can be adequately accounted for by linear analysis of the seismic-force-resisting system for the design spectrum, which is the elastic acceleration response spectrum reduced by the response modification factor, R. The effects of the horizontal component of ground motion perpendicular to the direction under consideration in the analysis, the vertical component of ground motion, and torsional motions of the structure are all considered in the same simplified approaches in the two procedures. The main difference between the two procedures lies in the distribution of the seismic lateral forces over the height of the building. In the modal analysis procedure, the distribution is based on properties of the natural vibration modes, which are determined from the mass and stiffness distribution. In the ELF procedure, the distribution is based on simplified formulas that are appropriate for regular structures as specified in Sec. 5.2.3. Otherwise, the two procedures are subject to the same limitations.

The simplifications inherent in the ELF procedure result in approximations that are likely to be inadequate if the lateral motions in two orthogonal directions and the torsional motion are strongly coupled. Such would be the case if the building were irregular in its plan configuration (see Sec. 4.3.2.2) or if it had a regular plan but its lower natural frequencies were nearly equal and the centers of mass and resistance were nearly coincident. The modal analysis method introduced in the 1997 *Provisions* includes a general model that is more appropriate for the analysis of such structures. It requires at least three degrees of freedom per floor—two for translational motion and one for torsional motion.

The methods of modal analysis can be generalized further to model the effect of diaphragm flexibility, soil-structure interaction, etc. In the most general form, the idealization would take the form of a large number of mass points, each with six degrees of freedom (three translational and three rotational) connected by generalized stiffness elements.

The ELF procedure (Sec. 5.2) and the response spectrum procedure are all likely to err systematically on the unsafe side if story strengths are distributed irregularly over height. This feature is likely to lead to concentration of ductility demand in a few stories of the building. The nonlinear static (or so-called pushover) procedure is a method to more accurately account for irregular strength distribution. However, it also has limitations and is not particularly applicable to tall structures or structures with relatively long fundamental periods of vibration.

The actual strength properties of the various components of a structure can be explicitly considered only by a nonlinear analysis of dynamic response by direct integration of the coupled equations of motion. This method has been used extensively in earthquake research studies of inelastic structural response. If the two lateral motions and the torsional motion are expected to be essentially uncoupled, it would be sufficient to include only one degree of freedom per floor, for motion in the direction along which the structure is being analyzed; otherwise at least three degrees of freedom per floor, two translational and one torsional, should be included. It should be recognized that the results of a nonlinear response history analysis of such mathematical structural models are only as good as are the models chosen to represent the structure vibrating at amplitudes of motion large enough to cause significant yielding during strong ground motions. Furthermore, reliable results can be achieved only by calculating the response to several ground motions—recorded accelerograms and/or simulated motions—and examining the statistics of response.

It is possible with presently available computer programs to perform two- and three-dimensional inelastic analyses of reasonably simple structures. The intent of such analyses could be to estimate the sequence in which components become inelastic and to indicate those components requiring strength adjustments so as to remain within the required ductility limits. It should be emphasized that with the present state of the art in analysis, there is no one method that can be applied to all types of structures. Further, the reliability of the analytical results are sensitive to:

1. The number and appropriateness of the input motion records,
2. The practical limitations of mathematical modeling including interacting effects of inelastic elements,
3. The nonlinear solution algorithms, and
4. The assumed hysteretic behavior of members.

Because of these sensitivities and limitations, the maximum base shear produced in an inelastic analysis should not be less than that required by Sec. 5.2.

The least rigorous analytical procedure that may be used in determining the design seismic forces and deformations in structures depends on the Seismic Design Category and the structural characteristics (in particular, regularity). Regularity is discussed in Sec. 4.3.2.

Except for structures assigned to Seismic Design Category A, the ELF procedure is the minimum level of analysis except that a more rigorous procedure is required for some Category D, E and F structures as identified in Table 4.4-1. The modal analysis procedure adequately addresses vertical irregularities of stiffness, mass, or geometry, as limited by the *Provisions*. Other irregularities must be carefully considered.

The basis for the ELF procedure and its limitations were discussed above. It is adequate for most regular structures; however, the designer may wish to employ a more rigorous procedure (see list of procedures at beginning of this section) for those regular structures where the ELF procedure may be inadequate. The ELF procedure is likely to be inadequate in the following cases:

1. Structures with irregular mass and stiffness properties in which case the simple equations for vertical distribution of lateral forces (Eq. 5.2-10 and 5.2-11) may lead to erroneous results;
2. Structures (regular or irregular) in which the lateral motions in two orthogonal directions and the torsional motion are strongly coupled; and

3. Structures with irregular distribution of story strengths leading to possible concentration of ductility demand in a few stories of the building.

In such cases, a more rigorous procedure that considers the dynamic behavior of the structure should be employed.

Structures with certain types of vertical irregularities may be analyzed as regular structures in accordance with the requirements of Sec. 5.2. These structures are generally referred to as setback structures. The following procedure may be used:

1. The base and tower portions of a building having a setback vertical configuration may be analyzed as indicated in (2) below if:
    a. The base portion and the tower portion, considered as separate structures, can be classified as regular and
    b. The stiffness of the top story of the base is at least five times that of the first story of the tower.

    When these conditions are not met, the building must be analyzed in accordance with Sec. 5.3.
2. The base and tower portions may be analyzed as separate structures in accordance with the following:
    a. The tower may be analyzed in accordance with the procedures in Sec. 5.2 with the base taken at the top of the base portion.
    b. The base portion then must be analyzed in accordance with the procedures in Sec. 5.2 using the height of the base portion of h_n and with the gravity load and seismic base shear forces of the tower portion acting at the top level of the base portion.

The design requirements in Sec. 5.3 include a simplified version of modal analysis that accounts for irregularity in mass and stiffness distribution over the height of the building. It would be adequate, in general, to use the ELF procedure for structures whose floor masses and cross-sectional areas and moments of inertia of structural members do not differ by more than 30 percent in adjacent floors and in adjacent stories.

For other structures, the following procedure should be used to determine whether the modal analysis procedures of Sec. 5.3 should be used:

1. Compute the story shears using the ELF procedure specified in Sec. 5.2.
2. On this basis, approximately dimension the structural members, and then compute the lateral displacements of the floor.
3. Replace h in Eq. 5.2-11 with these displacements, and recompute the lateral forces to obtain the revised story shears.
4. If at any story the recomputed story shear differs from the corresponding value as obtained from the procedures of Sec. 5.2 by more than 30 percent, the building should be analyzed using the procedure of Sec. 5.3. If the difference is less than this value, the building may be designed using the story shear obtained in the application of the present criterion and the procedures of Sec. 5.3 are not required.

Application of this procedure to these structures requires far less computational effort than the use of the response spectrum procedure of Sec. 5.3. In the majority of the structures, use of this procedure will determine that modal analysis need not be used and will also furnish a set of story shears that practically always lie much closer to the results of modal analysis than the results of the ELF procedure.

This procedure is equivalent to a single cycle of Newmark's method for calculation of the fundamental mode of vibration. It will detect both unusual shapes of the fundamental mode and excessively high influence of higher modes. Numerical studies have demonstrated that this procedure for determining whether modal analysis must be used will, in general, detect cases that truly should be analyzed dynamically; however, it generally will not indicate the need for dynamic analysis when such an analysis would not greatly improve accuracy.

5.2 EQUIVALENT LATERAL FORCE PROCEDURE

This section discusses the equivalent lateral force (ELF) procedure for seismic analysis of structures.

5.2.1 Seismic base shear. The heart of the ELF procedure is Eq. 5.2-1 for base shear, which gives the total seismic design force, V, in terms of two factors: a seismic response coefficient, C_s, and the seismic weight, W. The seismic response coefficient C_s, is obtained from Eq. 5.2-2 and 5.2-3 based on the design spectral response acceleration parameters, S_{DS} and S_{D1}. These acceleration parameters and the derivation of the response spectrum is discussed more fully in the *Commentary* for Chapter 3. The seismic weight is discussed in *Commentary* Sec. 1.5.1.

The base shear formula and the various factors contained therein were arrived at as explained below.

Elastic acceleration response spectrum. See the *Commentary* to Chapter 4 for a full discussion of the shape of the spectrum accounting for dynamic response amplification and the effect of site response.

Elastic design spectrum. The elastic acceleration response spectrum for earthquake motions has a descending branch for longer values of T, the period of vibration of the system, that varies roughly as a function of $1/T$. In previous editions of the *Provisions*, the actual response spectra that varied in a 1/T relationship were replaced with design spectra that varied in a $1/T^{2/3}$ relationship. This was intentionally done to provide added conservatism in the design of tall structures, as well as to account for the effects of higher mode participation. In the development of the 1997 *Provisions*, a special task force, known as the Seismic Design Procedures Group (SDPG), was convened to develop a method for using new seismic hazard maps, developed by the USGS in the *Provisions*. Whereas older seismic hazard maps provided an effective peak ground acceleration coefficient, C_a, and an effective peak velocity-related acceleration coefficient, C_v, the new maps directly provide parameters that correspond to points on the response spectrum. It was the recommendation of the SDPG that the true shape of the response spectrum, represented by a 1/T relationship, be used in the base shear equation. In order to maintain the added conservatism for tall and high occupancy structures, formerly provided by the design spectra which utilized a $1/T^{2/3}$ relationship, the 1997 *Provisions* adopted an occupancy importance factor I into the base shear equation. This I factor, which has a value of 1.25 for Seismic Use Group II structures and 1.5 for Seismic Use Group III structures has the effect of raising the design spectrum for taller, high occupancy structures, to levels comparable to those for which they were designed in previous editions of the *Provisions*.

Although the introduction of an occupancy importance factor in the 1997 edition adjusted the base shear to more conservative values for large buildings with higher occupancies, it did not address the issue of accounting for higher mode effects, which can be significant in longer period structures—those with fundamental modes of vibration significantly larger than the period T_s, at which the response spectrum changes from one of constant response acceleration (Eq. 5.2-2) to one of constant response velocity (Eq. 5.2-3).

Equation 5.2-3 could be modified to produce an estimate of base shear that is more consistent with the results predicted by elastic response spectrum methods. Some suggestions for such modifications may be found in Chopra (1995). However, it is important to note that even if the base shear equation were to simulate results of an elastic response spectrum analysis more accurately, most structures respond to design level ground shaking in an inelastic manner. This inelastic response results in different demands than are predicted by elastic analysis, regardless of how “exact” the analysis is. Inelastic response behavior in multistory buildings could be partially accounted for by other modifications to the seismic coefficient C_s. Specifically, the coefficient could be made larger to limit the ductility demand in multistory buildings to the same value as for single-degree-of-freedom systems. Results supporting such an approach may be found in (Chopra, 1995) and in (Nassar and Krawinkler, 1991).

The above notwithstanding, the equivalent lateral force procedure is intended to provide a relatively straightforward design approach where complex analyses, accurately accounting for dynamic and inelastic response effects, are not warranted. Rather than making the procedure more complex, so that it would be more appropriate for structures with significant higher mode response, in the 2000 edition of the *Provisions* application of this technique to structures assigned to Seismic Design Categories D, E, and F is limited to those where higher mode effects are not significant. Given the widespread use of computer-assisted analysis for major structures, it was felt that these limitations on the application of the equivalent lateral force procedure would not be burdensome. It should be noted that particularly for tall structures, the use of dynamic analysis methods will not only result in a more realistic characterization of the distribution of inertial forces in the structure, but may also result in reduced forces, particularly with regard to overturning demands. Therefore, use of a dynamic analysis method is recommended for such structures, regardless of the Seismic Design Category.

Historically, the ELF analytical approach has been limited in application in Seismic Design Categories D, E, and F to regular structures with heights of 240 ft (70 m) or less and irregular structures with heights of 100 ft (30 m) or less. Following recognition that the use of a base shear equation with a $1/T$ relationship underestimated the response of structures with significant higher mode participation, a change in the height limit for regular structures to 100 ft (30 m) was contemplated. However, the importance of higher mode participation in structural response is a function both of the structure's dynamic properties, which are dependent on height, mass and the stiffness of various lateral force resisting elements, and of the frequency content of the ground shaking, as represented by the response spectrum. Therefore, rather than continuing to use building height as the primary parameter used to control analysis procedures, it was decided to limit the application of the ELF to those structures in Seismic Design Categories D, E, and F having fundamental periods of response less than 3.5 times the period at which the response spectrum transitions from constant response acceleration to constant response velocity. This limit was selected based on comparisons of the base shear calculated by the ELF equations to that predicted by response spectrum analysis for structures of various periods on five different sites, representative of typical conditions in the eastern and western United States. For all 5 sites, it was determined that the ELF equations conservatively bound the results of a response spectrum analysis for structures having periods lower than the indicated amount.

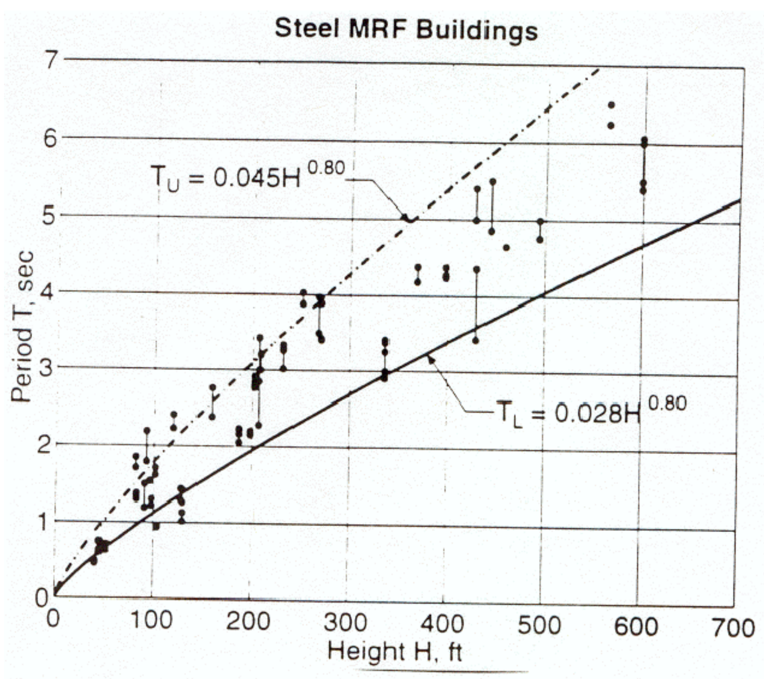

Figure C5.2-2 Measured building period for moment-resisting steel frame structures.

Response modification factor. The factor *R* in the denominator of Eq. 5.2-2 and 5.2-3 is an empirical response reduction factor intended to account for damping, overstrength, and the ductility inherent in the structural system at displacements great enough to surpass initial yield and approach the ultimate load displacement of the structural system. Thus, for a lightly damped building structure of brittle material that would be unable to tolerate any appreciable deformation beyond the elastic range, the factor *R* would be close to 1 (that is, no reduction from the linear elastic response would be allowed). At the other extreme, a heavily damped building structure with a very ductile structural system would be able to withstand deformations considerably in excess of initial yield and would, therefore, justify the assignment of a larger response reduction factor *R*. Table 4.3-1 in the *Provisions* stipulates *R* factors for different types of building systems using several different structural materials. The coefficient *R* ranges in value from a minimum of 1¼ for an unreinforced masonry bearing wall system to a maximum of 8 for a special moment frame system. The basis for the *R* factor values specified in Table 4.3-1 is explained in the *Commentary* to Sec. 4.2.1.

The effective value of *R* used in the base shear equation is adjusted by the occupancy importance factor *I*. The value of *I*, which ranges from 1 to 1.5, has the effect of reducing the amount of ductility the structure will be called on to provide at a given level of ground shaking. However, it must be recognized that added strength, by itself, is not adequate to provide for superior seismic performance in buildings with critical occupancies. Good connections and construction details, quality assurance procedures, and limitations on building deformation or drift are also important to significantly improve the capability for maintenance of function and safety in critical facilities and those with a high-density occupancy. Consequently, the reduction in the damage potential of critical facilities (Group III) is also handled by using more conservative drift controls (Sec. 4.5.1) and by providing special design and detailing requirements (Sec. 4.6) and materials limitations (Chapters 8 through 12).

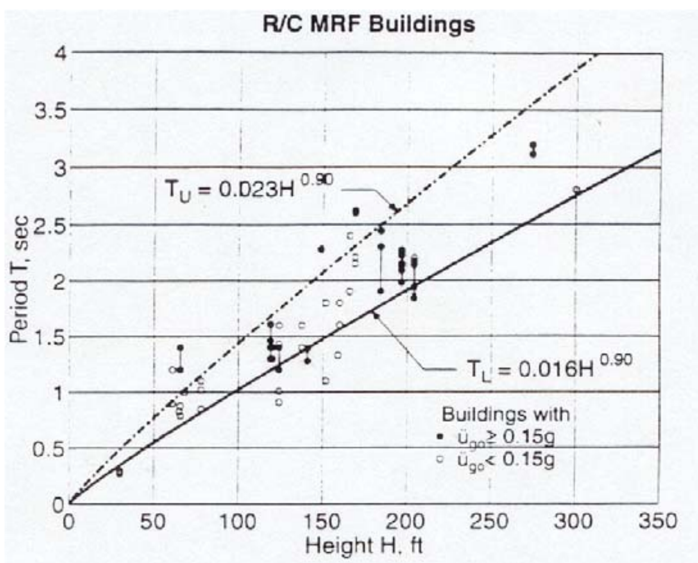

Figure C5.2-1 Measured building period for reinforced concrete frame structures.

5.2.2 Period determination. In the denominator of Eq. 5.2-3, *T* is the fundamental period of vibration of the structure. It is preferable that this be determined using modal analysis methods and the principles of structural mechanics. However, methods of structural mechanics cannot be employed to calculate the vibration period before a structure has been designed. Consequently, this section provides an approximate method that can be used to estimate the period, with minimal information available on the design. It is based on the use of simple formulas that involve only a general description of the type of structure (such as steel moment frame, concrete moment frame, shear wall system, braced frame) and overall dimensions (such as height and plan length) to estimate the period of vibration in order to calculate an initial base shear and proceed with a preliminary design.

It is advisable that this base shear and the corresponding value of *T* be conservative.

Even for final design, use of an unrealistically large value for *T* is unconservative. Thus, the value of *T* used in design should be smaller than the period calculated for the bare frame of the building. Equations 5.2-6, 5.2-7, and 5.2-8 for the approximate period T_a are therefore intended to provide conservative estimates of the fundamental period of vibration. An upper bound is placed on the value of *T* calculated using more exact methods, based on T_a and the factor C_u. The coefficient C_u is intended to reflect the likelihood that buildings in areas with lower lateral force requirements probably will be more flexible. Furthermore, it results in less dramatic changes from present practice in lower risk areas. It is generally accepted that the empirical equations for T_a are tailored to fit the type of construction common in areas with high lateral force requirements. It is unlikely that buildings in lower risk seismic areas would be designed to produce as high a drift level as allowed in the *Provisions* due to stability problems (*P*-delta) and wind requirements. Where the design of a structure is actually "controlled" by wind, the use of a large *T* will not really result in a lower design force; thus, use of this approach in high-wind regions should not result in unsafe design.

Taking the seismic base shear to vary as a function of 1/*T* and assuming that the lateral forces are distributed linearly over the height and that the deflections are controlled by drift limitations, a simple calculation of the period of vibration by Rayleigh's method leads to the conclusion that the vibration period of moment resisting frame structures varies roughly as $h_n^{3/4}$ where h_n equals the total height of the building as defined elsewhere. Based on this, for many years Eq. 5.2-6 appeared in the *Provisions* in the form:

$$T_a = C_t h_n^{3/4}$$

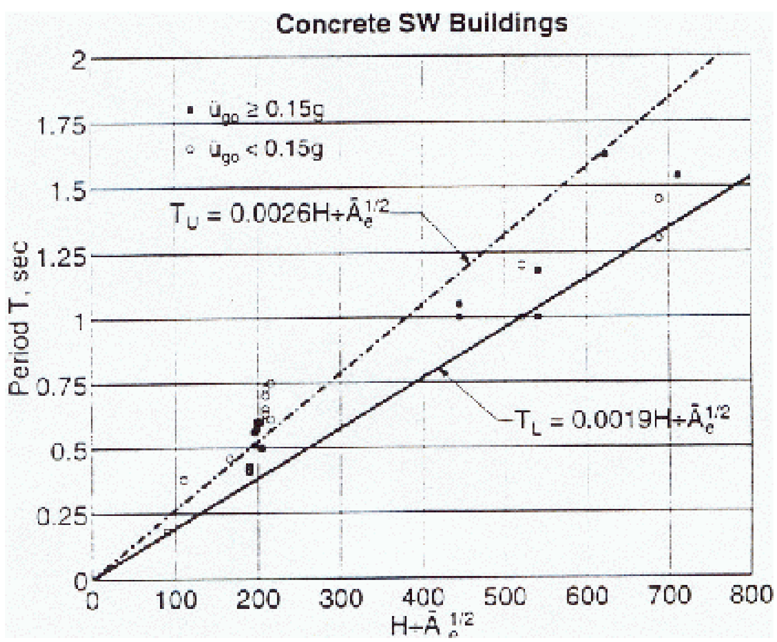

Figure C5.2-3 Measured building period for concrete shear wall structures.

A large number of strong motion instruments have been placed in buildings located within zones of high seismic activity by the U.S. Geological Survey and the California Division of Mines and Geology. Over the past several years, this has allowed the response to strong ground shaking for a significant number of these buildings to be recorded and the fundamental period of vibration of the buildings to be calculated.

Figures C5.2-1, C5.2-2, and C5.2-3, respectively, show plots of these data as a function of building height for three classes of structures. Figure C5.2-1 shows the data for moment-resisting concrete frame buildings; Figure C5.2-2, for moment-resisting steel frame buildings; and Figure C5.2-3, for concrete shear wall buildings. Also shown in these figures are equations for lines that envelop the data within approximately a standard deviation above and below the mean.

For the 2000 *Provisions*, Eq. 5.2-6 is revised into a more general form allowing the statistical fits of the data shown in the figures to be used directly. The values of the coefficient C_r and the exponent x given in Table 5.2-2 for these moment-resisting frame structures represent the lower bound (mean minus one standard deviation) fits to the data shown in Figures C5.2-1 and C5.2-2, respectively, for steel and concrete moment frames. Although updated data were available for concrete shear wall structures, these data do not fit well with an equation of the form of Eq. 5.2-6. This is because the period of shear wall buildings is highly dependent not only on the height of the structure but also on the amount of shear wall present in the building. Analytical evaluations performed by Chopra and Goel (1997 and 1998) indicate that equations of the form of Eq. 5.2-8 and 5.2-9 provide a reasonably good fit to the data. However, the form of these equations is somewhat complex. Therefore, the simpler form of Eq. 5.2-6 contained in earlier editions of the *Provisions* was retained with the newer, more accurate formulation presented as an alternative.

Updated data for other classes of construction were not available. As a result, the C_r and x values for other types of construction shown in Table 5.2-2 are values largely based on limited data obtained from the 1971 San Fernando earthquake that have been used in past editions of the *Provisions*. The optional use of $T = 0.1N$ (Eq. 5.2-7) is an approximation for low to moderate height frames that has long been in use.

In earlier editions of the *Provisions*, the C_u coefficient varied from a value of 1.2 in zones of high seismicity to a value of 1.7 in zones of low seismicity. The data presented in Figures C5.2-1, C5.2-2, and C5.2-3 permit direct evaluation of the upper bound on period as a function of the lower bound, given by Eq. 5.2-6. This data indicates that in zones of high seismicity, the ratio of the upper to lower bound may more properly be taken as a value of about 1.4. Therefore, in the 2000 *Provisions*, the values in Table 5.2-1 were revised to reflect this data in zones of high seismicity while retaining the somewhat subjective values contained in earlier editions for the zones of lower seismicity.

For exceptionally stiff or light buildings, the calculated T for the seismic-force-resisting system may be significantly shorter than T_a calculated by Eq. 5.2-6. For such buildings, it is recommended that the period value T be used in lieu of T_a for calculating the seismic response coefficient, C_s.

Although the approximate methods of Sec. 5.2.2.1 can be used to determine a period for the design of structures, the fundamental period of vibration of the seismic-force-resisting system should be calculated according to established methods of mechanics. Computer programs are available for such calculations. One method of calculating the period, probably as convenient as any, is the use of the following formula based on Rayleigh's method:

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^{n} w_i \delta_i^2}{g \sum_{i=1}^{n} F_i \delta_i}} \qquad \text{(C5.2-1)}$$

where:

F_i = the seismic lateral force at Level i,

w_i = the seismic weight assigned in Level i,

δ_i = the static lateral displacement at Level i due to the forces F_i computed on a linear elastic basis, and

g = is the acceleration due to gravity.

The calculated period increases with an increase in flexibility of the structure because the δ term in the Rayleigh formula appears to the second power in the numerator but to only the first power in the denominator. Thus, if one ignores the contribution of nonstructural elements to the stiffness of the structure in calculating the deflections δ, the deflections are exaggerated and the calculated period is lengthened, leading to a decrease in the seismic response coefficient C_s and, therefore, a decrease in the design force. Nonstructural elements participate in the behavior of the structure even though the designer may not rely on them to contribute any strength or stiffness to the structure. To ignore them in calculating the period is to err on the unconservative side. The limitation of C_uT_a is imposed as a safeguard.

5.2.3 Vertical distribution of seismic forces. The distribution of lateral forces over the height of a structure is generally quite complex because these forces are the result of superposition of a number of natural modes of vibration. The relative contributions of these vibration modes to the total forces depends on a number of factors including the shape of the earthquake response spectrum, the natural periods of vibration of the structure, and the shapes of vibration modes that, in turn, depend on the distribution of mass and stiffness over the height. The basis of this method is discussed below. In structures having only minor irregularity of mass or stiffness over the height, the accuracy of the lateral force distribution as given by Eq. 5.2-11 is much improved by the procedure described in the last portion of Sec. 5.1 of this commentary. The lateral force at each level, x, due to response in the first (fundamental) natural mode of vibration is given by Eq. C5.2-2 as follows:

$$f_{x1} = V_1 \left[\frac{w_x \phi_{x1}}{\sum_{i=1}^{n} w_1 \phi_{i1}} \right] \qquad \text{(C5.2-2)}$$

where:

V_1 = the contribution of this mode to the base shear,

w_i = the weight lumped at the ith level, and

ϕ_i = the amplitude of the first mode at the i^{th} level.

This is the same as Eq. 5.3-7 in Sec. 5.3.5 of the *Provisions*, but it is specialized for the first mode. If V_1 is replaced by the total base shear, V, this equation becomes identical to Eq. 5.2-11 with $k = 1$ if the first mode shape is a straight line and with $k = 2$ if the first mode shape is a parabola with its vertex at the base.

It is well known that the influence of modes of vibration higher than the fundamental mode is small in the earthquake response of short period structures and that, in regular structures, the fundamental vibration mode departs little from a straight line. This, along with the matters discussed above, provides the basis for Eq. 5.2-11 with $k = 1$ for structures having a fundamental vibration period of 0.5 seconds or less.

It has been demonstrated that although the earthquake response of long period structures is primarily due to the fundamental natural mode of vibration, the influence of higher modes of vibration can be significant and, in regular structures, the fundamental vibration mode lies approximately between a straight line and a parabola with the vertex at the base. Thus, Eq. 5.2-11 with $k = 2$ is appropriate for structures having a fundamental period of vibration of 2.5 seconds or longer. Linear variation of k between 1 at a 0.5 second period and 2 at a 2.5 seconds period provides the simplest possible transition between the two extreme values.

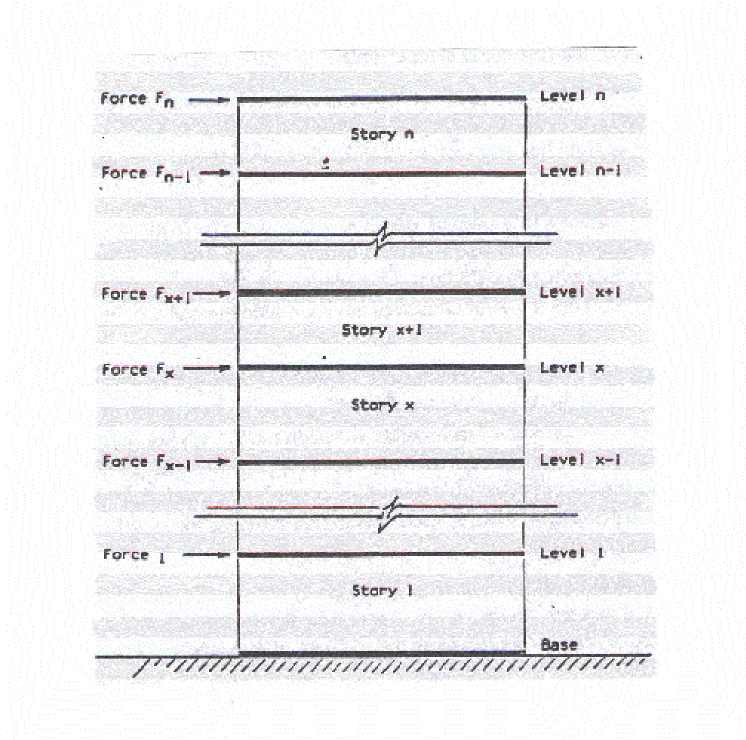

Figure C5.2-4 Description of story and level.
The shear at Story *x* (V_x) is the sum of all the lateral forces at and above Story *x* (F_x through F_n).

5.2.4 Horizontal shear distribution. The story shear in any story is the sum of the lateral forces acting at all levels above that story. Story *x* is the story immediately below Level *x* (Figure C5.2-4). Reasonable and consistent assumptions regarding the stiffness of concrete and masonry elements may be used for analysis in distributing the shear force to such elements connected by a horizontal diaphragm. Similarly, the stiffness of moment or braced frames will establish the distribution of the story shear to the vertical resisting elements in that story.

5.2.4.1 and 5.2.4.2 Inherent and accidental torsion. The torsional moment to be considered in the design of elements in a story consists of two parts:

1. M_t, the moment due to eccentricity between centers of mass and resistance for that story, which is computed as the story shear times the eccentricity perpendicular to the direction of applied earthquake forces.
2. M_{ta}, commonly referred to as "accidental torsion," which is computed as the story shear times the "accidental eccentricity," equal to 5 percent of the dimension of the structure (in the story under consideration) perpendicular to the direction of the applied earthquake forces.

Computation of M_{ta} in this manner is equivalent to the procedure in Sec. 5.2.4.2 which implies that the dimension of the structure is the dimension in the story where the torsional moment is being computed and that all the masses above that story should be assumed to be displaced in the same direction at one time (for example, first, all of them to the left and, then, to the right).

Dynamic analyses assuming linear behavior indicate that the torsional moment due to eccentricity between centers of mass and resistance may significantly exceed M_t (Newmark and Rosenblueth, 1971). However, such dynamic magnification is not included in the *Provisions*, partly because its significance is not well understood for structures designed to deform well beyond the range of linear behavior.

The torsional moment M_t calculated in accordance with this provision would be zero in those stories where centers of mass and resistance coincide. However, during vibration of the structure, torsional moments would be induced in such stories due to eccentricities between centers of mass and resistance in other stories. To account for such effects, it is recommended that the torsional moment in any story be no smaller than the following two values (Newmark and Rosenblueth, 1971):

1. The story shear times one-half of the maximum of the computed eccentricities in all stories below the one being analyzed and
2. One-half of the maximum of the computed torsional moments for all stories above.

Accidental torsion is intended to cover the effects of several factors that have not been explicitly considered in the *Provisions*. These factors include the rotational component of ground motion about a vertical axis; unforeseeable differences between computed and actual values of stiffness, yield strengths, and dead-load masses; and unforeseeable unfavorable distributions of dead- and live-load masses.

The way in which the story shears and the effects of torsional moments are distributed to the vertical elements of the seismic-force-resisting system depends on the stiffness of the diaphragms relative to vertical elements of the system.

Where the diaphragm stiffness in its own plane is sufficiently high relative to the stiffness of the vertical components of the system, the diaphragm may be assumed to be indefinitely rigid for purposes of this section. Then, in accordance with compatibility and equilibrium requirements, the shear in any story is to be distributed among the vertical components in proportion to their contributions to the lateral stiffness of the story while the story torsional moment produces additional shears in these components that are proportional to their contributions to the torsional stiffness of the story about its center of resistance. This contribution of any component is the product of its lateral stiffness and the square of its distance to the center of resistance of the story. Alternatively, the story shears and torsional moments may be distributed on the basis of a three-dimensional analysis of the structure, consistent with the assumption of linear behavior.

Where the diaphragm in its own plane is very flexible relative to the vertical components, each vertical component acts nearly independently of the rest. The story shear should be distributed to the vertical components considering these to be rigid supports. Analysis of the diaphragm acting as a continuous horizontal beam or truss on rigid supports leads to the distribution of shears. Because the properties of the beam or truss may not be accurately computed, the shears in vertical elements should not be taken to be less than those based on "tributary areas." Accidental torsion may be accounted for by adjusting the position of the horizontal force with respect to the supporting vertical elements.

There are some common situations where it is obvious that the diaphragm can be assumed to be either rigid or very flexible in its own plane for purposes of distributing story shear and considering torsional moments. For example, a solid monolithic reinforced concrete slab, square or nearly square in plan, in a structure with slender moment resisting frames may be regarded as rigid. A large plywood diaphragm with widely spaced and long, low masonry walls may be regarded as very flexible. In intermediate situations, the design forces should be based on an analysis that explicitly considers diaphragm deformations and satisfies equilibrium and compatibility requirements. Alternatively, the design forces could be based on the envelope of the two sets of forces resulting from both extreme assumptions regarding the diaphragms—rigid or very flexible.

Where the horizontal diaphragm is not continuous and the elements perpendicular to the direction of motion are ignored, the story shear can be distributed to the vertical components based on their tributary areas.

5.2.4.3 Dynamic amplification of torsion. There are indications that the 5 percent accidental eccentricity may be too small in some structures since they may develop torsional dynamic instability. Some examples are the upper stories of tall structures having little or no nominal eccentricity, those structures where the calculations of relative stiffnesses of various elements are particularly uncertain (such as those that depend largely on masonry walls for lateral force resistance or those that depend on vertical elements made of different materials), and nominally symmetrical structures that utilize core elements alone for seismic resistance or that behave essentially like elastic nonlinear systems (for example, some prestressed concrete frames). The amplification factor for torsionally irregular structures (Eq. 5.2-13) was introduced in the 1988 Edition as an attempt to account for some of these problems in a controlled and rational way.

5.2.5 Overturning. This section requires that the structure be designed to resist overturning moments statically consistent with the design story shears. In the 1997 and earlier editions of the *Provisions*, the overturning moment was modified by a factor, τ, to account, in an approximate manner, for the effects of higher mode response in taller structures. In the 2000 edition of the *Provisions*, the equivalent lateral force procedure was limited in application in Seismic Design Categories D, E, and F to structures that do not have significant higher mode participation. As a result it was possible to simplify the design procedure by eliminating the τ factor. Under this new approach tall structures in Seismic Design Categories B and C designed using the equivalent lateral force procedure will be designed for somewhat larger overturning demands than under past editions of the *Provisions*. This conservatism was accepted as an inducement for designers of such structures to use a more appropriate dynamic analysis procedure.

In the design of the foundation, the overturning moment calculated at the foundation-soil interface may be reduced to 75 percent of the calculated value using Eq. 5.2-14. This is appropriate because a slight uplifting of one edge of the foundation during vibration leads to reduction in the overturning moment and because such behavior does not normally cause structural distress.

5.2.6 Drift determination and *P*-delta effects. This section defines the design story drift as the difference of the deflections, δ_x, at the top and bottom of the story under consideration. The deflections, δ_x, are determined by multiplying the deflections, δ_{xe} (determined from an elastic analysis), by the deflection amplification factor, C_d, given in Table 4.3-1. The elastic analysis is to be made for the seismic-force-resisting system using the prescribed seismic design forces and considering the structure to be fixed at the base. Stiffnesses other than those of the seismic-force-resisting system should not be included since they may not be reliable at higher inelastic strain levels.

The deflections are to be determined by combining the effects of joint rotation of members, shear deformations between floors, the axial deformations of the overall lateral resisting elements, and the shear and flexural deformations of shear walls and braced frames. The deflections are determined initially on the basis of the distribution of lateral forces stipulated in Sec. 5.2.3. For frame structures, the axial deformations from bending effects, although contributing to the overall structural distortion, may or may not affect the story-to-story drift; however, they are to be considered. Centerline dimensions between the frame elements often are used for analysis, but clear-span dimensions with consideration of joint panel zone deformation also may be used.

For determining compliance with the story drift limitation of Sec. 4.5.1, the deflections, δ_x, may be calculated as indicated above for the seismic-force-resisting system and design forces corresponding to the fundamental period of the structure, T (calculated without the limit $T \leq C_u T_a$ specified in Sec. 5.2.2), may be used. The same model of the seismic-force-resisting system used in determining the deflections must be used for determining T. The waiver does not pertain to the calculation of drifts for determining *P*-delta effects on member forces, overturning moments, etc. If the *P*-delta effects determined in Sec. 5.2.6.2 are significant, the design story drift must be increased by the resulting incremental factor.

The *P*-delta effects in a given story are due to the eccentricity of the gravity load above that story. If the story drift due to the lateral forces prescribed in Sec. 5.2.3 were Δ, the bending moments in the story would be augmented by an amount equal to Δ times the gravity load above the story. The ratio of the *P*-delta moment to the lateral force story moment is designated as a stability coefficient, θ, in Eq. 5.2-16. If the stability coefficient θ is less than 0.10 for every story, the *P*-delta effects on story shears and moments and member forces may be ignored. If, however, the stability coefficient θ exceeds 0.10 for any story, the *P*-delta effects on story drifts, shears, member forces, etc., for the whole structure must be determined by a rational analysis.

An acceptable *P*-delta analysis, based upon elastic stability theory, is as follows:

1. Compute for each story the *P*-delta amplification factor, $a_d = \theta/(1 - \theta)$. a_d takes into account the multiplier effect due to the initial story drift leading to another increment of drift that would lead to yet another increment, etc. Thus, both the effective shear in the story and the computed eccentricity would be augmented by a factor $1 + \theta + \theta^2 + \theta^3$ ..., which is $1/(1 - \theta)$ or $(1 + a_d)$.
2. Multiply the story shear, V_x, in each story by the factor $(1 + a_d)$ for that story and recompute the story shears, overturning moments, and other seismic force effects corresponding to these augmented story shears.

This procedure is applicable to planar structures and, with some extension, to three-dimensional structures. Methods exist for incorporating two- and three-dimensional *P*-delta effects into computer analyses that do not explicitly include such effects (Rutenburg, 1985). Many programs explicitly include *P*-delta effects. A mathematical description of the method employed by several popular programs is given by Wilson and Habibullah (1987).

The *P*-delta procedure cited above effectively checks the static stability of a structure based on its initial stiffness. Since the inception of this procedure with ATC 3-06, however, there has been some debate regarding its accuracy. This debate stems from the intuitive notion that the structure's secant stiffness would more accurately represent inelastic *P*-delta effects. Given the additional uncertainty of the effect of dynamic response on *P*-delta behavior and the (apparent) observation that instability-related failures rarely occur in real structures, the *P*-delta requirements remained as originally written until revised for the 1991 Edition.

There was increasing evidence that the use of elastic stiffness in determining *theoretical P*-delta response is unconservative. Given a study carried out by Bernal (1987), it was argued that *P*-delta amplifiers should be based on secant stiffness and that, in other words, the C_d term in Eq. 5.2-16 should be deleted. However, since Bernal's study was based on the inelastic response of single-degree-of-freedom, elastic-perfectly plastic systems, significant uncertainties existed regarding the extrapolation of the concepts to the complex hysteretic behavior of multi-degree-of-freedom systems.

Another problem with accepting a *P*-delta procedure based on secant stiffness is that design forces would be greatly increased. For example, consider an ordinary moment frame of steel with a C_d of 4.0 and an elastic stability coefficient θ of 0.15. The amplifier for this structure would be 1.0/0.85 = 1.18 according to the 1988 Edition of the *Provisions*. If the *P*-delta effects were based on secant stiffness, however, the stability coefficient would increase to 0.60 and the amplifier would become 1.0/0.4 = 2.50. This example illustrates that there could be an extreme impact on the requirements if a change were implemented that incorporated *P*-delta amplifiers based on static secant stiffness response.

There was, however, some justification for retaining the *P*-delta amplifier as based on elastic stiffness. This justification was the apparent lack of stability-related failures. The reasons for the lack of observed failures included:

1. Many structures display strength well above the strength implied by code-level design forces (see Figure C4.2-3). This overstrength likely protects structures from stability-related failures.
2. The likelihood of a failure due to instability decreases with increased intensity of expected ground-shaking. This is due to the fact that the stiffness of most structures designed for extreme ground motion is significantly greater than the stiffness of the same structure designed for lower intensity shaking or for wind. Since damaging, low-intensity earthquakes are somewhat rare, there would be little observable damage.

Due to the lack of stability-related failures, therefore, recent editions of the *Provisions* regarding *P*-delta amplifiers have remained from the 1991 Editions.

The 1991 Edition introduced a requirement that the computed stability coefficient, θ, not exceed 0.25 or $0.5/\beta C_d$, where βC_d is an adjusted ductility demand that takes into account the fact that the seismic strength demand may be somewhat less than the code strength supplied. The adjusted ductility demand is not intended to incorporate overstrength beyond that computed by the means available in Chapters 8 through 14 of the *Provisions*.

The purpose of this requirement is to protect structures from the possibility of stability failures triggered by post-earthquake residual deformation. The danger of such failures is real and may not be eliminated by apparently available overstrength. This is particularly true of structures designed in regions of lower seismicity.

The computation of θ_{max}, which, in turn, is based on βC_d, requires the computation of story strength supply and story strength demand. Story strength demand is simply the seismic design shear for the story under consideration. The story strength supply may be computed as the shear in the story that occurs simultaneously with the attainment of the development of first significant yield of the overall structure. To compute first significant yield, the structure should be loaded with a seismic force pattern similar to that used to compute seismic story strength demand. A simple and conservative procedure is to compute the ratio of demand to strength for each member of the seismic-force-resisting system in a particular story and then use the largest such ratio as β. For a structure otherwise in conformance with the *Provisions*, taking β equal to 1.0 is obviously conservative.

The principal reason for inclusion of β is to allow for a more equitable analysis of those structures in which substantial extra strength is provided, whether as a result of added stiffness for drift control, for code-required wind resistance, or simply a feature of other aspects of the design. Some structures inherently possess more strength than required, but instability is not typically a concern for such structures. For many flexible structures, the proportions of the structural members are controlled by the drift requirements rather than the strength requirements; consequently, β is less than 1.0 because the members provided are larger and stronger than required. This has the effect of reducing the inelastic component of total seismic drift and, thus, β is placed as a factor on C_d.

Accurate evaluation of β would require consideration of all pertinent load combinations to find the maximum value of seismic load effect demand to seismic load effect capacity in each and every member. A conservative simplification is to divide the total demand with seismic included by the total capacity; this covers all load combinations in which dead and live effects add to seismic. If a member is controlled by a load combination where dead load counteracts seismic, to be correctly computed, the ratio β must be based only on the seismic component, not the total; note that the vertical load P in the P-delta computation would be less in such a circumstance and, therefore, θ would be less. The importance of the counteracting load combination does have to be considered, but it rarely controls instability.

5.3 RESPONSE SPECTRUM PROCEDURE

Modal analysis (Newmark and Rosenblueth, 1971; Clough and Penzien, 1975; Thomson, 1965; Wiegel, 1970) is applicable for calculating the linear response of complex, multi-degree-of-freedom structures and is based on the fact that the response is the superposition of the responses of individual natural modes of vibration, each mode responding with its own particular pattern of deformation (the mode shape), with its own frequency (the modal frequency), and with its own modal damping. The response of the structure, therefore, can be modeled by the response of a number of single-degree-of-freedom oscillators with properties chosen to be representative of the mode and the degree to which the mode is excited by the earthquake motion. For certain types of damping, this representation is mathematically exact and, for structures, numerous full-scale tests and analyses of earthquake response of structures have shown that the use of modal analysis, with viscously damped single-degree-of-freedom oscillators describing the response of the structural modes, is an accurate approximation for analysis of linear response.

Modal analysis is useful in design. The ELF procedure of Sec. 5.2 is simply a first mode application of this technique, which assumes all of the structure’s mass is active in the first mode. The purpose of modal analysis is to obtain the maximum response of the structure in each of its important modes, which are then summed in an appropriate manner. This maximum modal response can be expressed in several ways. For the *Provisions*, it was decided that the modal forces and their distributions over the structure should be given primary emphasis to highlight the similarity to the equivalent static methods traditionally used in building codes (the SEAOC recommendations and the *UBC*) and the ELF procedure in Sec. 5.2. Thus, the coefficient C_{sm} in Eq. 5.3-3 and the distribution equations, Eq. 5.3-1 and 5.3-2, are the counterparts of Eq. 5.2-10 and 5.2-11. This correspondence helps clarify the fact that the simplified modal analysis contained in Sec. 5.3 is simply an attempt to specify the equivalent lateral forces on a structure in a way that directly reflects the individual dynamic characteristics of the structure. Once the story shears and other response variables for each of the important modes are determined and combined to produce design values, the design values are used in basically the same manner as the equivalent lateral forces given in Sec. 5.2.

5.3.2 Modes. This section defines the number of modes to be used in the analysis. For many structures, including low-rise structures and structures of moderate height, three modes of vibration in each direction are nearly always sufficient to determine design values of the earthquake response of the structure. For high-rise structures, however, more than three modes may be required to adequately determine the forces for design. This section provides a simple rule that the combined participating mass of all modes considered in the analysis should be equal to or greater than 90 percent of the effective total mass in each of two orthogonal horizontal directions.

5.3.3 Modal properties. Natural periods of vibration are required for each of the modes used in the subsequent calculations. These are needed to determine the modal coefficients C_{sm} is Sec. 5.3.4. Because the periods of the modes contemplated in these requirements are those associated with moderately large, but still essentially linear, structural response, the period calculations should include only those elements that are effective at these amplitudes. Such periods may be longer than those obtained from a small-amplitude test of the structure when completed or the response to small earthquake motions because of the stiffening effects of nonstructural and architectural components of the structure at small amplitudes. During response to strong ground-shaking, however, measured responses of structures have shown that the periods lengthen, indicating the loss of the stiffness contributed by those components.

There exists a wide variety of methods for calculation of natural periods and associated mode shapes, and no one particular method is required by the *Provisions*. It is essential, however, that the method used be one based on generally accepted principles of mechanics such as those given in well known textbooks on structural dynamics and vibrations (Clough and Penzien, 1975; Newmark and Rosenblueth, 1971; Thomson, 1965; Wiegel, 1970). Although it is expected that in many cases computer programs, whose accuracy and reliability are documented and widely recognized, will be used to calculate the required natural periods and associated mode shapes, their use is not required.

5.3.4 Modal base shear. A central feature of modal analysis is that the earthquake response is considered as a combination of the independent responses of the structure vibrating in each of its important modes. As the structure vibrates back and forth in a particular mode at the associated period, it experiences maximum values of base shear, story drifts, floor displacements, base (overturning) moments, etc. In this section, the base shear in the m^{th} mode is specified as the product of the modal seismic coefficient C_{sm} and the effective weight W_m for the mode. The coefficient C_{sm} is determined for each mode from Eq. 5.3-3 using the spectral acceleration S_{am} at the associated period of the mode, T_m, in addition to the R, which is discussed elsewhere in the *Commentary*. An exception to this procedure occurs for higher modes of those structures that have periods shorter than 0.3 second and that are founded on soils of Site Class D, E, or F. For such modes, Eq. 5.3-4 is used. Equation 5.3-4 gives values ranging from $0.4S_{DS}/R$ for very short periods to S_{DS}/R for $T_m = 0.3$. Comparing these values to the limiting values of C_s of S_{DS}/R for Site Class D, it is seen that the use of Eq. 5.3-4, when applicable, reduces the modal base shear. This is an approximation introduced in consideration of the conservatism embodied in using the spectral shape specified in Sec. 3.3.4. The spectral shape so defined is a conservative approximation to average spectra that are known to first ascend, level off, and then decay as period increases. The design spectrum defined in Sec. 3.3.4 is somewhat more conservative. For Site Classes A, B, and C, the ascending portion of the spectra is completed at or below periods of 0.1 to 0.2 second. On the other hand, for soft soils the ascent may not be completed until a larger period is reached. Equation 5.3-4 is then a replacement for the spectral shape for Site Classes D, E and F and short periods that is more consistent with spectra for measured accelerations. It was introduced because it was judged unnecessarily conservative to use Eq. 3.3-5 for modal analysis of structures assigned to Site Classes D, E, and F. The effective modal seismic weight given in Eq. 5.3-2 can be interpreted as specifying the portion of the weight of the structure that participates in the vibration of each mode. It is noted that Eq. 5.3-2 gives values of W_m that are independent of how the modes are normalized.

The final equation of this section, Eq. 5.3-5, is to be used if a modal period exceeds 4 seconds. It can be seen that Eq. 5.3-5 and 5.3-3 coincide at T_m equal to 4 seconds so that the effect of using Eq. 5.3-5 is to provide a more rapid decrease in C_{sm} as a function of the known characteristics of earthquake response spectra at intermediate and long periods. At intermediate periods, the average velocity spectrum of strong earthquake motions from large (magnitude 6.5 and larger) earthquakes is approximately constant, which implies that C_{sm} should decrease as $1/T_m$. For very long periods, the average displacement spectrum of strong earthquake motions becomes constant which implies that C_{sm}, a form of acceleration spectrum, should decay as $1/T_m^2$. The period at which the displacement response spectrum becomes constant depends on the size of the earthquake, being larger for great earthquakes, and a representative period of 4 seconds was chosen to make the transition.

5.3.5 Modal forces, deflections, and drifts. This section specifies the forces and displacements associated with each of the important modes of response.

Modal forces at each level are given by Eq. 5.3-6 and 5.3-7 and are expressed in terms of the seismic weight assigned to the floor, the mode shape, and the modal base shear V_m. In applying the forces F_{xm} to the structure, the direction of the forces is controlled by the algebraic sign of f_{xm}. Hence, the modal forces for the fundamental mode will all act in the same direction, but modal forces for the second and higher modes will change direction as one moves up the structure. The form of Eq. 5.3-6 is somewhat different from that usually employed in standard references and shows clearly the relation between the modal forces and the modal base shear. It, therefore, is a convenient form for calculation and highlights the similarity to Eq. 5.2-10 in the ELF procedure.

The modal deflections at each level are specified by Eq. 5.3-8 and 5.3-9. These are the displacements caused by the modal forces F_{xm} considered as static forces and are representative of the maximum amplitudes of modal response for the essentially elastic motions envisioned within the concept of the seismic response modification coefficient R. If the mode under consideration dominates the earthquake response, the modal deflection under the strongest motion contemplated by the *Provisions* can be estimated by multiplying by the deflection amplification factor C_d. It should be noted that δ_{xm} is proportional to ϕ_{xm} (this can be shown with algebraic substitution for F_{xm} in Eq. 5.3-9) and will therefore change direction up and down the structure for the higher modes.

5.3.6 Modal story shears and moments. This section merely specifies that the forces of Eq. 5.3-6 should be used to calculate the shears and moments for each mode under consideration. In essence, the forces from Eq. 5.3-6 are applied to each mass, and linear static methods are used to calculate story shears and story overturning moments. The base shear that results from the calculation should agree with computed using Eq. 5.3-1.

5.3.7 Design values. This section specifies the manner in which the values of story shear, moment, and drift and the deflection at each level are to be combined. The method used, in which the design value is the square root of the sum of the squares of the modal quantities, was selected for its simplicity and its wide familiarity (Clough and Penzien, 1975; Newmark and Rosenblueth, 1971; Wiegel, 1970). In general, it gives satisfactory results, but it is not always a conservative predictor of the earthquake response inasmuch as more adverse combinations of modal quantities than are given by this method of combination can occur. The most common instance where combination by use of the square root of the sum of the squares is unconservative occurs when two modes have very nearly the same natural period. In this case, the responses are highly correlated and the designer should consider combining the modal quantities more conservatively (Newmark and Rosenblueth, 1971). The complete quadratic combination (CQC) technique provides somewhat better results than the square-root-of-the-sum-of-the-squares method for the case of closely spaced modes.

This section also limits the reduction of base shear that can be achieved by modal analysis compared to use of the ELF procedure. Some reduction, where it occurs, is thought to be justified because the modal analysis gives a somewhat more accurate representation of the earthquake response. Some limit to the reduction permitted as a result of the calculation of longer natural periods is necessary because the actual periods of vibration may not be as long, even at moderately large amplitudes of motion, due to the stiffening effects of structural elements not a part of the seismic-force-resisting system and of nonstructural components. The limit is imposed by comparison to 85 percent of the base shear value computed using the ELF procedure. Where modal analysis predicts response quantities corresponding to a total base shear less than 85 percent of that which is computed using the ELF procedure, all response results must be scaled up to that level. Where modal analysis predicts response quantities in excess of those predicted by the ELF procedure, this is likely the result of significant higher mode participation and reduction to the values obtained from the ELF procedure is not permitted.

5.3.8 Horizontal shear distribution. This section requires that the design story shears calculated in Sec. 5.3.6 and the torsional moments prescribed in Sec. 5.2.4 be distributed to the vertical elements of the seismic resisting system as specified in Sec. 5.2.4 and as elaborated on in the corresponding section of this commentary.

5.3.9 Foundation overturning. Because story moments are calculated mode by mode (properly recognizing that the direction of forces F_{xm} is controlled by the algebraic sign of f_{xm}) and then combined to obtain the design values of story moments, there is no reason for reducing these design moments. This is in contrast with reductions permitted in overturning moments calculated from equivalent lateral forces in the analysis procedures of Sec. 5.2 (see Sec. 5.2.5 of this commentary). However, in the design of the foundation, the overturning moment calculated at the foundation-soil interface may be reduced by 10 percent for the reasons mentioned in Sec. 5.2.5 of this commentary.

5.3.10 *P*-delta effects. Sec. 5.2.6 of this commentary applies to this section. In addition, to obtain the story drifts when using the modal analysis procedure of Sec. 5.3, the story drift for each mode should be determined independently for each story. The story drift should not be determined from the differential of combined lateral structural deflections since this latter procedure will tend to mask the higher mode effects in longer period structures.

5.4 LINEAR RESPONSE HISTORY PROCEDURE

Linear response history analysis, also commonly known as time history analysis, is a numerically involved technique in which the response of a structural model to a specific earthquake ground motion accelerogram is determined through a process of numerical integration of the equations of motion. The ground shaking accelerogram, or record, is digitized into a series of small time steps, typically on the order of 1/100th of a second or smaller. Starting at the initial time step, a finite difference solution, or other numerical integration algorithm is followed to allow the calculation of the displacements of each node in the model and the forces in each element of model for each time step of the record. For even small structural models, this requires thousands of calculations and produces tens of thousands of data points. Clearly, such a calculation procedure can be performed only with the aid of high speed computers. However, even with the use of such computers, which are now commonly available, interpretation of the voluminous data that results from such analysis is tedious.

The principal advantages of response history analysis, as opposed to response spectrum analysis, is that response history analysis provides a time dependent history of the response of the structure to a specific ground motion, allowing calculation of path dependent effects such as damping and also providing information on the stress and deformation state of the structure throughout the period of response. A response spectrum analysis, however, indicates only the maximum response quantities and does not indicate when during the period of response these occur, or how response of different portions of the structure is phased relative to that of other portions. Response history analyses are highly dependent on the characteristics of the individual ground shaking records and subtle changes in these records can lead to significant differences with regard to the predicted response of the structure. This is why, when response history analyses are used in the design process, it is necessary to run a suite of ground motion records. The use of multiple records in the analyses allows observation of the difference in response, resulting from differences in record characteristics. As a minimum, the *Provisions* require that suites of ground motions include at least three different records. However, suites containing larger numbers of records are preferable, since when more records are run, it is more likely that the differing response possibilities for different ground motion characteristics are observed. In order to encourage the use of larger suites, the *Provisions* require that when a suite contains fewer than seven records, the maximum values of the predicted response parameters be used as the design values. When seven or more records are used, then mean values of the response parameters may be used. This can lead to a substantial reduction in design forces and displacements and typically will justify the use of larger suites of records.

Where possible, ground motion records should be scaled from actual recorded earthquake ground motions with characteristics (earthquake magnitude, distance from causative fault, and site soil conditions) similar to those which control the design earthquake for the site. Since only a limited number of actual recordings are available for such purposes, the use of synthetic records is permitted and may often be required.

The extra complexity and cost inherent in the use of response history analysis rather than modal response spectrum analysis is seldom justified. As a result this procedure is rarely used in the design process. One exception is for the design of structures with energy dissipation systems comprising linear viscous dampers. Linear response history analysis can be used to predict the response of structures with such systems, while modal response spectrum analysis cannot.

5.5 NONLINEAR RESPONSE HISTORY PROCEDURE

This method of analysis is very similar to linear response history analysis, described in Sec. 5.4, except that the mathematical model is formulated in such a way that the stiffness and even connectivity of the elements can be directly modified based on the deformation state of the structure. This permits the effects of element yielding, buckling, and other nonlinear behavior on structural response to be directly accounted for in the analysis. It also permits the evaluation of such nonlinear behaviors as foundation rocking, opening and closing of gaps, and nonlinear viscous and hysteric damping. Potentially, this ability to directly account for these various nonlinearities can permit nonlinear response history analysis to provide very accurate evaluations of the response of the structure to strong ground motion. However, this accuracy can seldom be achieved in practice. This is partially because currently available nonlinear models for different elements can only approximate the behavior of real structural elements. Another limit on the accuracy of this approach is the fact that minor deviations in ground motion, such as those described in Sec. 5.4, or even in element hysteretic behavior, can result in significant differences in predicted response. For these reasons, when nonlinear response history analysis is used in the design process, suites of ground motion time histories must be considered, as described in Sec. 5.4. It may also be appropriate to perform sensitivity studies, in which the assumed hysteretic properties of elements are allowed to vary, within expected bounds, to allow evaluation of the effects of such uncertainties on predicted response.

Application of nonlinear response history analysis to even the simplest structures requires large, high speed computers and complex computer software that has been specifically developed for this purpose. Several software packages have been in use for this purpose in universities for a number of years. These include the DRAIN family of programs and also the IDARC and IDARST family of programs. However, these programs have largely been viewed as experimental and are not generally accompanied by the same level of documentation and quality assurance typically found with commercially available software packages typically used in design offices. Although commercial software capable of performing nonlinear response history analyses has been available for several years, the use of these packages has generally been limited to complex aerospace, mechanical, and industrial applications.

As a result of this, nonlinear response history analysis has mostly been used as a research (rather than design) tool until very recently. With the increasing adoption of base isolation and energy dissipation technologies in the structural design process, however, the need to apply this analysis technique in the design office has increased, creating a demand for more commercially available software. In response to this demand, several vendors of commercial structural analysis software have modified their analysis programs to include limited nonlinear capability including the ability to model base isolation bearings, viscous dampers, and friction dampers. Some of these programs also have a limited library of other nonlinear elements including beam and truss elements. Such software provides the design office with the ability to begin to practically implement nonlinear response history analysis on design projects. However, such software is still limited, and it is expected that it will be some years before design offices can routinely expect to utilize this technique in the design of complex structures.

5.5.3.1 Member strength. Nonlinear response history analysis is primarily a deformation-based procedure, in which the amount of nonlinear deformation imposed on elements by response to earthquake ground shaking is predicted. As a result, when this analysis method is employed, there is no general need to evaluate the strength demand (forces) imposed on individual elements of the structure. Instead, the adequacy of the individual elements to withstand the imposed deformation demands is directly evaluated, under the requirements of Sec. 5.5.3.2. The exception to this is the requirement to evaluate brittle elements, the failure of which could result in structural collapse, for the forces predicted by the analysis. These elements are identified in the *Provisions* through the requirement that they be evaluated for earthquake forces using the seismic effects defined in Sec. 4.2.2.2. That section requires that forces predicted by elastic analysis be amplified by a factor, Ω_0, to account in an approximate manner for the actual maximum force that can be delivered to the element, considering the inelastic behavior of the structure. Since nonlinear response history analysis does not use a response modification factor, as do elastic analysis approaches, and directly accounts for inelastic structural behavior, there is no need to further increase the forces by this factor. Instead the forces predicted by the analysis are used directly in the evaluation of the elements for adequacy under Sec. 4.2.2.2.

5.5.4 Design review. The provisions for design using linear methods of analysis including the equivalent lateral force technique of Sec. 5.2 and the modal response spectrum analysis technique of Sec. 5.3, are highly prescriptive. They limit the modeling assumptions that can be employed as well as the minimum strength and stiffness the structure must possess. Further, the methods used in linear analysis have become standardized in practice such that it is unlikely that different designers using the same technique to analyze the same structure will produce substantially different results. However, when nonlinear analytical methods are employed to predict the structure's strength and its deformation under load, many of these prescriptive provisions are no longer applicable. Further, as these methods are currently not widely employed by the profession, the standardization that has occurred for linear methods of analysis has not yet been developed for these techniques. As a result analysis has not yet been developed for these techniques, and the designer using such methods must employ a significant amount of independent judgment in developing appropriate analytical models, performing the analysis, and interpreting the results to confirm the adequacy of a design. Since relatively minor changes in the assumptions used in performing a nonlinear structural analysis can significantly affect the results obtained from such an analysis, it is imperative that the assumptions used be appropriate. The *Provisions* require that designs employing nonlinear analysis methods be subjected to independent design review in order to provide a level of assurance that the independent judgment applied by the designer when using these methods is appropriate and compatible with that which would be made by other competent practitioners.

5.6 SOIL-STRUCTURE INTERACTION EFFECTS

5.6.1 General

Statement of the problem. Fundamental to the design requirements presented in Sec. 5.2 and 5.3 is the assumption that the motion experienced by the base of a structure during an earthquake is the same as the "free-field" ground motion, a term that refers to the motion that would occur at the level of the foundation if no structure was present. This assumption implies that the foundation-soil system underlying the structure is rigid and, hence, represents a "fixed-base" condition. Strictly speaking, this assumption never holds in practice. For structures supported on a deformable soil, the foundation motion generally is different from the free-field motion and may include an important rocking component in addition to a lateral or translational component. The rocking component, and soil-structure interaction effects in general, tend to be most significant for laterally stiff structures such as buildings with shear walls, particularly those located on soft soils. For convenience, in what follows the response of a structure supported on a deformable foundation-soil system will be denoted as the "flexible-base" response.

A flexibly supported structure also differs from a rigidly supported structure in that a substantial part of its vibrational energy may be dissipated into the supporting medium by radiation of waves and by hysteretic action in the soil. The importance of the latter factor increases with increasing intensity of ground-shaking. There is, of course, no counterpart of this effect of energy dissipation in a rigidly supported structure.

The effects of soil-structure interaction accounted for in Sec. 5.6 represent the difference in the flexible-base and fixed-base responses of the structure. This difference depends on the properties of the structure and the supporting medium as well as the characteristics of the free-field ground motion.

The interaction effects accounted for in Sec. 5.6 should not be confused with "site effects," which refer to the fact that the characteristics of the free-field ground motion induced by a dynamic event at a given site are functions of the properties and geological features of the subsurface soil and rock. The interaction effects, on the other hand, refer to the fact that the dynamic response of a structure built on that site depends, in addition, on the interrelationship of the structural characteristics and the properties of the local underlying soil deposits. The site effects are reflected in the values of the seismic coefficients employed in Sec. 5.2 and 5.3 and are accounted for only implicitly in Sec. 5.6.

Possible approaches to the problem. Two different approaches may be used to assess the effects of soil-structure interaction. The first involves modifying the stipulated free-field design ground motion, evaluating the response of the given structure to the modified motion of the foundation, and solving simultaneously with additional equations that define the motion of the coupled system, whereas the second involves modifying the dynamic properties of the structure and evaluating the response of the modified structure to the prescribed free-field ground motion (Jennings and Bielak, 1973; Veletsos, 1977). When properly implemented, both approaches lead to equivalent results. However, the second approach, involving the use of the free-field ground motion, is more convenient for design purposes and provides the basis of the requirements presented in Sec. 5.6.

Characteristics of interaction. The interaction effects in the approach used here are expressed by an increase in the fundamental natural period of the structure and a change (usually an increase) in its effective damping.

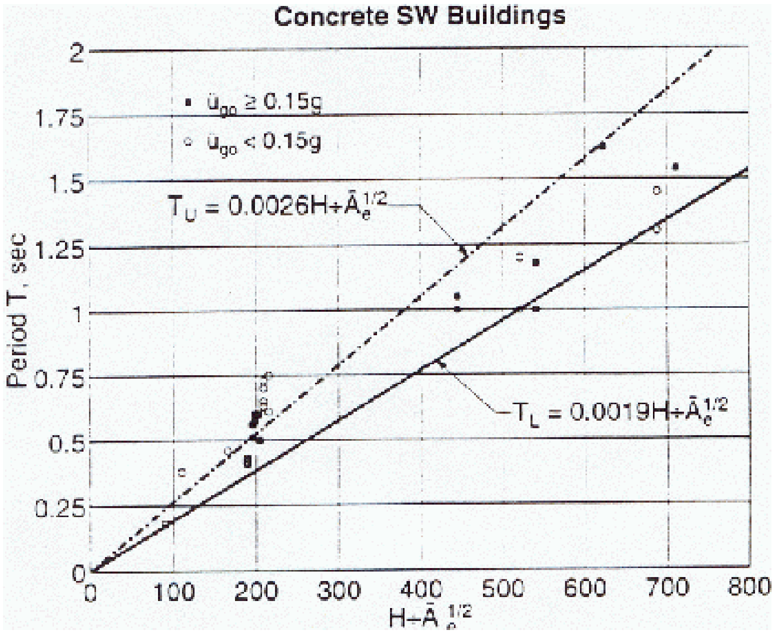

Figure C5.2-3 Measured building period for concrete shear wall structures.

The increase in period results from the flexibility of the foundation soil whereas the change in damping results mainly from the effects of energy dissipation in the soil due to radiation and material damping.

These statements can be clarified by comparing the responses of rigidly and elastically supported systems subjected to a harmonic excitation of the base.

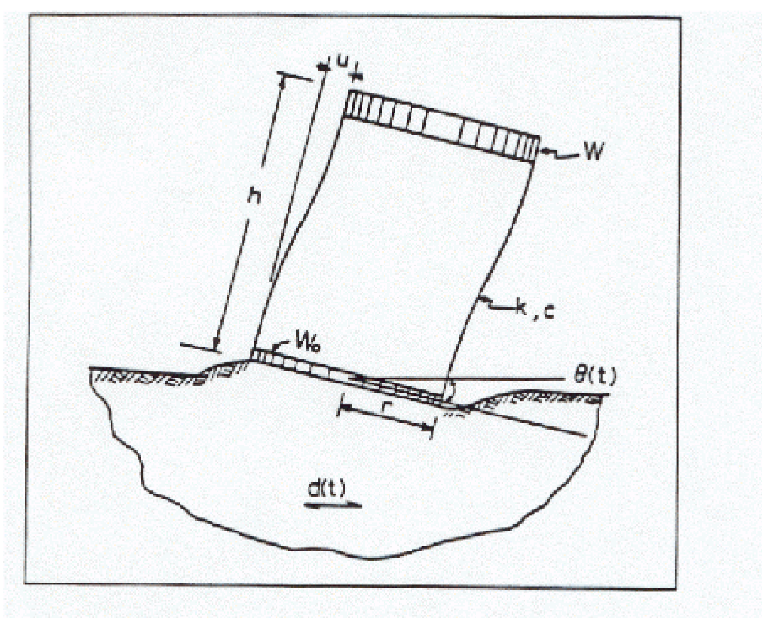

Figure C5.6-1 Simple system investigated.

Consider a linear structure of weight W, lateral stiffness k, and coefficient of viscous damping c (shown in Figure C5.6-1) and assume that it is supported by a foundation of weight W_o at the surface of a homogeneous, elastic halfspace.

The foundation mat is idealized as a rigid circular plate of negligible thickness bonded to the supporting medium, and the columns of the structure are considered to be weightless and axially inextensible. Both the foundation weight and the weight of the structure are assumed to be uniformly distributed over circular areas of radius r. The base excitation is specified by the free-field motion of the ground surface. This is taken as a horizontally directed, simple harmonic motion with a period T_o and an acceleration amplitude a_m.

The configuration of this system, which has three degrees of freedom when flexibly supported and a single degree of freedom when fixed at the base, is specified by the lateral displacement and rotation of the foundation, y and θ, and by the displacement of the top of the structure, u, relative to its base. The system may be viewed either as the direct model of a one-story structural frame or, more generally, as a model of a multistory, multimode structure that responds as a single-degree-of-freedom system in its fixed-base condition. In the latter case, h must be interpreted as the distance from the base to the centroid of the inertia forces associated with the fundamental mode of vibration of the fixed-base structure and W, k, and c must be interpreted as its generalized or effective weight, stiffness, and damping coefficient, respectively. The relevant expressions for these quantities are given below.

The solid lines in Figures C5.6-2 and C5.6-3 represent response spectra for the steady-state amplitude of the total shear in the columns of the system considered in Figure C5.6-1. Two different values of h/r and several different values of the relative flexibility parameter for the soil and the structure, ϕ_o, are considered. The latter parameter is defined by the equation $\delta_0 = \frac{h}{v_s T}$ in which h is the height of the structure as previously indicated, v_s is the velocity of shear wave propagation in the halfspace, and T is the fixed-base natural period of the structure. A value of $\phi = 0$ corresponds to a rigidly supported structure. The results in Figures C5.6-2 and C5.6-3 are displayed in a dimensionless form, with the abscissa representing the ratio of the period of the excitation, T_o, to the fixed-base natural period of the system, T, and the ordinate representing the ratio of the amplitude of the actual base shear, V, to the amplitude of the base shear induced in an infinitely stiff, rigidly supported structure.

The latter quantity is given by the product ma_m, in which $m = W/g$, g is the acceleration due to gravity, and a_m is the acceleration amplitude of the free-field ground motion. The inclined scales on the left represent the deformation amplitude of the superstructure, u, normalized with respect to the displacement amplitude of the free-field ground motion $d_m = \frac{a_m T_0^2}{4\pi^2}$.

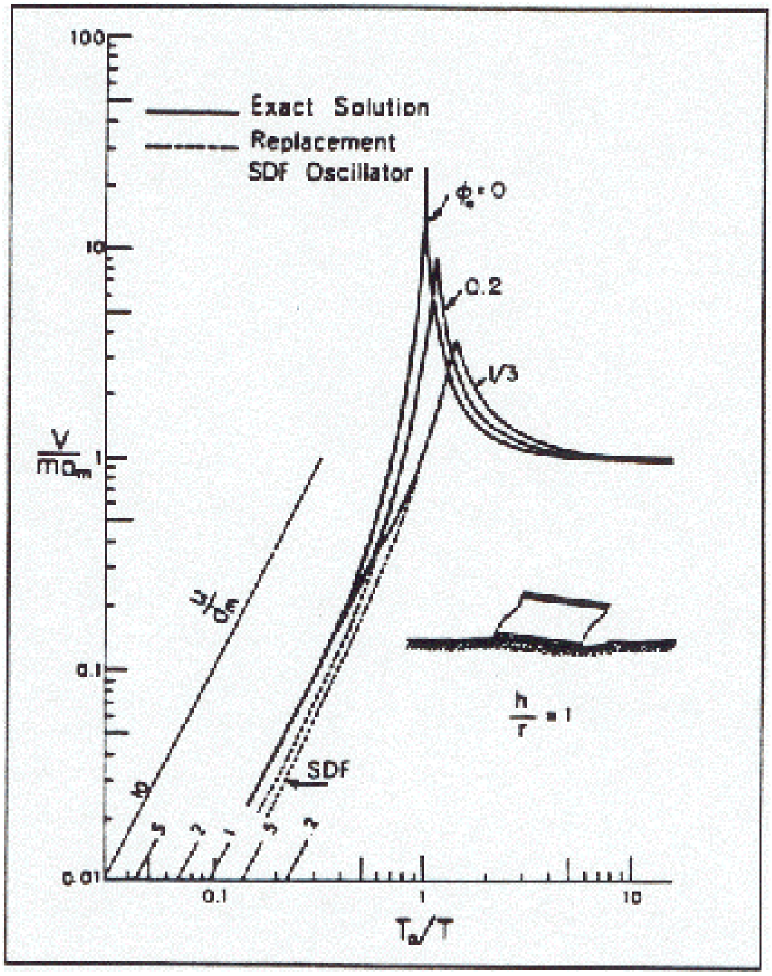

Figure C5.6-2 Response spectra for systems with *h*/*r* = 1 (Veletsos and Meek, 1974).

The damping of the structure in its fixed-base condition, β, is considered to be 2 percent of the critical value, and the additional parameters needed to characterize completely these solutions are identified in Veletsos and Meek (1974), from which these figures have been reproduced.

Comparison of the results presented in these figures reveals that the effects of soil-structure interaction are most strikingly reflected in a shift of the peak of the response spectrum to the right and a change in the magnitude of the peak. These changes, which are particularly prominent for taller structures and more flexible soils (increasing values of ϕ_0), can conveniently be expressed by an increase in the natural period of the system over its fixed-base value and by a change in its damping factor.

Also shown in these figures in dotted lines are response spectra for single-degree-of-freedom (SDF) oscillators, the natural period and damping of which have been adjusted so that the absolute maximum (resonant) value of the base shear and the associated period are in each case identical to those of the actual interacting systems. The base motion for the replacement oscillator is considered to be the same as the free-field ground motion. With the properties of the replacement SDF oscillator determined in this manner, it is important to note that the response spectra for the actual and the replacement systems are in excellent agreement over wide ranges of the exciting period on both sides of the resonant peak.

In the context of Fourier analysis, an earthquake motion may be viewed as the result of superposition of harmonic motions of different periods and amplitudes. Inasmuch as the components of the excitation with periods close to the resonant period are likely to be the dominant contributors to the response, the maximum responses of the actual system and of the replacement oscillator can be expected to be in satisfactory agreement for earthquake ground motions as well. This expectation has been confirmed by the results of comprehensive comparative studies (Veletsos, 1977; Veletsos and Meek, 1974; Veletsos and Nair, 1975; Jennings and Bielak, 1973).

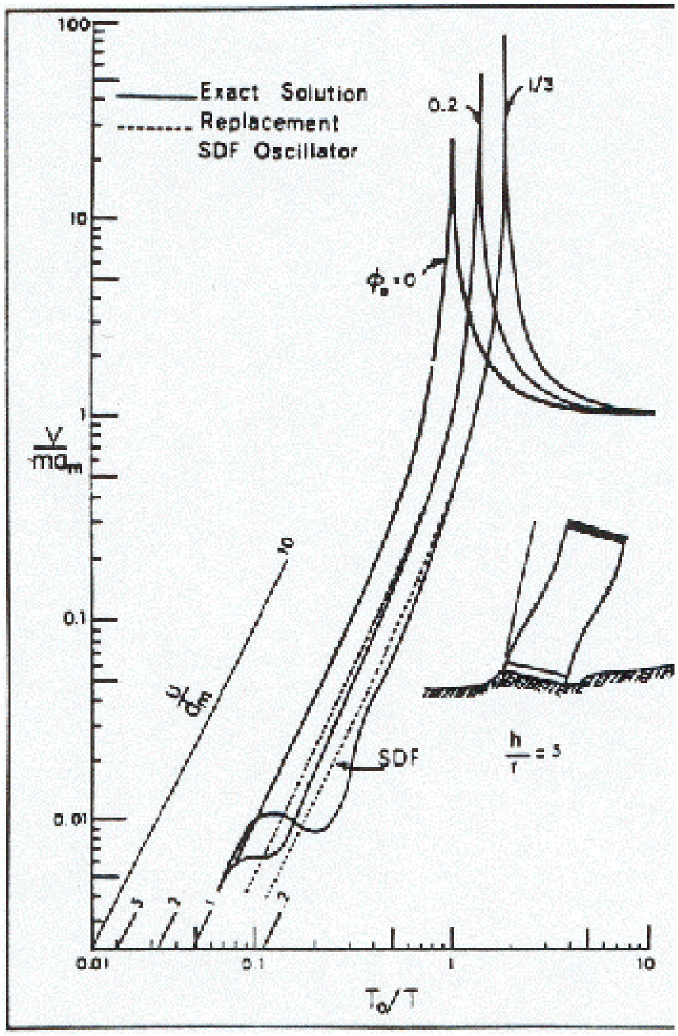

Figure C5.6-3 Response spectra for systems with *h*/*r* = 5 (Veletsos and Meek, 1974).

It follows that, to the degree of approximation involved in the representation of the actual system by the replacement SDF oscillator, the effects of interaction on maximum response may be expressed by an increase in the fundamental natural period of the fixed-base system and by a change in its damping value. In the following sections, the natural period of replacement oscillator is denoted by $\tilde{T}$ and the associated damping factor by $\tilde{\beta}$. These quantities will also be referred to as the effective natural period and the effective damping factor of the interacting system. The relationships between $\tilde{T}$ and T and between $\tilde{\beta}$ and β are considered in Sec. 5.6.2.1.1 and 5.6.2.1.2.

Basis of provisions and assumptions. Current knowledge of the effects of soil-structure interactions is derived mainly from studies of systems of the type referred to above in which the foundation is idealized as a rigid mat. For foundations of this type, both surface-supported and embedded structures resting on uniform as well as layered soil deposits have been investigated (Bielak, 1975; Chopra and Gutierrez, 1974; Jennings and Bielak, 1973; Liu and Fagel, 1971; Parmelee et al., 1969; Roesset et al., 1973; Veletsos, 1977; Veletsos and Meek, 1974; Veletsos and Nair, 1975). However, the results of such studies may be of limited applicability for foundation systems consisting of individual spread footings or deep foundations (piles or drilled shafts) not interconnected with grade beams or a mat. The requirements presented in Sec. 5.6 for the latter cases represent the best interpretation and judgment of the developers of the requirements regarding the current state of knowledge.

Fundamental to these requirements is the assumption that the structure and the underlying soil are bonded and remain so throughout the period of ground-shaking. It is further assumed that there is no soil instability or large foundation settlements. The design of the foundation in a manner to ensure satisfactory soil performance (for example, to avoid soil instability and settlement associated with the compaction and liquefaction of loose granular soils), is beyond the scope of Sec. 5.6. Finally, no account is taken of the interaction effects among neighboring structures.

Nature of interaction effects. Depending on the characteristics of the structure and the ground motion under consideration, soil-structure interaction may increase, decrease, or have no effect on the magnitudes of the maximum forces induced in the structure itself (Bielak, 1975; Jennings and Bielak, 1973; Veletsos, 1977; Veletsos and Meek, 1974; Veletsos and Nair, 1975). However, for the conditions stipulated in the development of the requirements for rigidly supported structures presented in Sec. 5.2 and 5.3, soil-structure interaction will reduce the design values of the base shear and moment from the levels applicable to a fixed-base condition. These forces therefore can be evaluated conservatively without the adjustments recommended in Sec. 5.6.

Because of the influence of foundation rocking, however, the horizontal displacements relative to the base of the elastically supported structure may be larger than those of the corresponding fixed-base structure, and this may increase both the required spacing between structures and the secondary design forces associated with the P-delta effects. Such increases generally are small for frame structures, but can be significant for shear wall structures.

Scope. Two procedures are used to incorporate effects of the soil-structure interaction. The first is an extension of the equivalent lateral force procedure presented in Sec. 5.2 and involves the use of equivalent lateral static forces. The second is an extension of the simplified modal analysis procedure presented in Sec. 5.3. In the latter approach, the earthquake-induced effects are expressed as a linear combination of terms, the number of which is equal to the number of stories involved. Other more complex procedures also may be used, and these are outlined briefly at the end of this commentary on Sec. 5.6. However, it is believed that the more involved procedures are justified only for unusual structures and when the results of the specified simpler approaches have revealed that the interaction effects are indeed of definite consequence in the design.

5.6.2 Equivalent lateral force procedure. This procedure is similar to that used in the older SEAOC recommendations except that it incorporates several improvements (see Sec. 5.2 of this commentary). In effect, the procedure considers the response of the structure in its fundamental mode of vibration and accounts for the contributions of the higher modes implicitly through the choice of the effective weight of the structure and the vertical distribution of the lateral forces. The effects of soil-structure interaction are accounted for on the assumption that they influence only the contribution of the fundamental mode of vibration. For structures, this assumption has been found to be adequate (Bielak, 1976; Jennings and Bielak, 1973; Veletsos, 1977).

5.6.2.1 Base shear. With the effects of soil-structure interaction neglected, the base shear is defined by Eq. 5.2-1, $V = C_sW$, in which W is the total seismic weight (as specified in Sec. 5.2.1) and C_s is the dimensionless seismic response coefficient (as defined in Sec. 5.2.1.1). This term depends on the level of seismic hazard under consideration, the properties of the site, and the characteristics of the structure itself.

The latter characteristics include the rigidly supported fundamental natural period of the structure, T, the associated damping factor, β, and the degree of permissible inelastic deformation. The damping factor does not appear explicitly in Sec. 5.2.1.1 because a constant value of $\beta = 0.05$ has been used for all structures for which the interaction effects are negligible. The degree of permissible inelastic action is reflected in the choice of the reduction factor, R. It is convenient to rewrite Eq. 5.2-1 in the form:

$$V = C_s(T,\beta)\bar{W} + C_s(T,\beta)[W - \bar{W}] \quad \text{(C5.6-1)}$$

where $\bar{W}$ represents the generalized or effective weight of the structure when vibrating in its fundamental natural mode. The terms in parentheses are used to emphasize the fact that C_s depends upon both T and β. The relationship between $\bar{W}$ and W is given below. The first term on the right side of Eq. C5.6-1 approximates the contribution of the fundamental mode of vibration whereas the second term approximates the contributions of the higher natural modes. Inasmuch as soil-structure interaction may be considered to affect only the contribution of the fundamental mode and inasmuch as this effect can be expressed by changes in the fundamental natural period and the associated damping of the system, the base shear for the interacting system, $\tilde{V}$, may be stated (in a form analogous to Eq. C5.6-1) as follows:

$$\tilde{V} = C_s(\tilde{T},\tilde{\beta})\bar{W} + C_s(T,\beta)[W - \bar{W}] \quad \text{(C5.6-2)}$$

The value of C_s in the first part of this equation should be evaluated for the natural period and damping of the elastically supported system, $\tilde{T}$ and $\tilde{\beta}$, respectively, and the value of C_s in the second term part should be evaluated for the corresponding quantities of the rigidly supported system, T and β.

Before proceeding with the evaluation of the coefficients C_s in Eq. C5.6-2, it is desirable to rewrite this formula in the same form as Eq. 5.6-1. Making use of Eq. 5.2-1 and rearranging terms, the following expression for the reduction in the base shear is obtained:

$$\Delta V = \left[C_s(T,\beta) - C_s(\tilde{T},\tilde{\beta})\right]\bar{W} \quad \text{(C5.6-3)}$$

Within the ranges of natural period and damping that are of interest in studies of structural response, the values of C_s corresponding to two different damping values but the same natural period (T), are related approximately as follows:

$$C_s(\tilde{T},\tilde{\beta}) = C_s(\tilde{T},\beta)\left(\frac{\beta}{\tilde{\beta}}\right)^{0.4} \quad \text{(C5.6-4)}$$

This expression, which appears to have been first proposed in Arias and Husid (1962), is in good agreement with the results of studies of earthquake response spectra for systems having different damping values (Newmark et al., 1973).

Substitution of Eq. C5.6-4 in Eq. C5.6-3 leads to:

$$\Delta V = \left[C_s(T,\beta) - C_s(\tilde{T},\beta)\left(\frac{\beta}{\tilde{\beta}}\right)^{0.4}\right]\bar{W} \quad \text{(C5.6-5)}$$

where both values of C_s are now for the damping factor of the rigidly supported system and may be evaluated from Eq. 5.2-2 and 5.2-3. If the terms corresponding to the periods T and $\tilde{T}$ are denoted more simply as C_s and $\tilde{C}_s$, respectively, and if the damping factor β is taken as 0.05, Eq. C5.6-5 reduces to Eq. 5.6-2.

Note that $\tilde{C}_s$ in Eq. 5.6-2 is smaller than or equal to C_s because Eq. 5.2-3 is a nonincreasing function of the natural period and $\tilde{T}$ is greater than or equal to T. Furthermore, since the minimum value of $\tilde{\beta}$ is taken as $\tilde{\beta} = \beta = 0.05$ (see statement following Eq. 5.6-10), the shear reduction ΔV is a non-negative quantity. It follows that the design value of the base shear for the elastically supported structure cannot be greater than that for the associated rigid-base structure.

The effective weight of the structure, $\bar{W}$, is defined by Eq. 5.3-2, in which ϕ_{im} should be interpreted as the displacement amplitude of the i^{th} floor when the structure is vibrating in its fixed-base fundamental natural mode. It should be clear that the ratio $\bar{W}/W$ depends on the detailed characteristics of the structure. A constant value of $\bar{W} = 0.7\ W$ is recommended in the interest of simplicity and because it is a good approximation for typical structures. As an example, it is noted that for a tall structure for which the weight is uniformly distributed along the height and for which the fundamental natural mode increases linearly from the base to the top, the exact value of $\bar{W} = 0.7\ W$. Naturally, when the full weight of the structure is concentrated at a single level, $\bar{W}$ should be taken equal to W.

The maximum permissible reduction in base shear due to the effects of soil-structure interaction is set at 30 percent of the value calculated for a rigid-base condition. It is expected, however, that this limit will control only infrequently and that the calculated reduction, in most cases, will be less.

5.6.2.1.1 Effective building period. Equation 5.6-3 for the effective natural period of the elastically supported structure, $\tilde{T}$, is determined from analyses in which the superstructure is presumed to respond in its fixed-base fundamental mode and the foundation weight is considered to be negligible in comparison to the weight of the superstructure (Jennings and Bielak, 1973; Veletsos and Meek, 1974). The first term under the radical represents the period of the fixed-base structure. The first portion of the second term represents the contribution to $\tilde{T}$ of the translational flexibility of the foundation, and the last portion represents the contribution of the corresponding rocking flexibility. The quantities $\bar{k}$ and $\bar{h}$ represent, respectively, the effective stiffness and effective height of the structure, and K_y and K_θ represent the translational and rocking stiffnesses of the foundation.

Equation 5.6-4 for the structural stiffness, $\bar{k}$, is deduced from the well known expression for the natural period of the fixed-base system:

$$T = 2\pi\sqrt{\left(\frac{1}{g}\right)\left(\frac{\bar{W}}{\bar{k}}\right)} \tag{C5.6-6}$$

The effective height, $\bar{h}$, is defined by Eq. 5.6-13, in which ϕ_{i1} has the same meaning as the quantity ϕ_{im} in Eq. 5.3-2 when $m = 1$. In the interest of simplicity and consistency with the approximation used in the definition of $\bar{W}$, however, a constant value of $\bar{h} = 0.7h_n$ is recommended where h_n is the total height of the structure. This value represents a good approximation for typical structures. As an example, it is noted that for tall structures for which the fundamental natural mode increases linearly with height, the exact value of $\bar{h}$ is $2/3h_n$. Naturally, when the gravity load of the structure is effectively concentrated at a single level, h_n must be taken as equal to the distance from the base to the level of weight concentration.

Foundation stiffnesses depend on the geometry of the foundation-soil contact area, the properties of the soil beneath the foundation, and the characteristics of the foundation motion. Most of the available information on this subject is derived from analytical studies of the response of harmonically excited rigid circular foundations, and it is desirable to begin with a brief review of these results.

For circular mat foundations supported at the surface of a homogeneous halfspace, stiffnesses K_y and K_θ are given by:

$$K_y = \left[\frac{8\alpha_y}{2-\nu}\right]Gr \tag{C5.6-7}$$

and

$$K_\theta = \left[\frac{8\alpha_\theta}{3(1-v)}\right] Gr^3 \qquad \text{(C5.6-8)}$$

where r is the radius of the foundation; G is the shear modulus of the halfspace; v is its Poisson's ratio; and α_y and α_θ are dimensionless coefficients that depend on the period of the excitation, the dimensions of the foundation, and the properties of the supporting medium (Luco, 1974; Veletsos and Verbic, 1974; Veletsos and Wei, 1971). The shear modulus is related to the shear wave velocity, v_s, by the formula:

$$G = \frac{\gamma v_s^2}{g} \qquad \text{(C5.6-9)}$$

in which γ is the unit weight of the material. The values of G, v_s, and v should be interpreted as average values for the region of the soil that is affected by the forces acting on the foundation and should correspond to the conditions developed during the design earthquake. The evaluation of these quantities is considered further in subsequent sections. For statically loaded foundations, the stiffness coefficients α_y and α_θ are unity, and Eq. C5.6-7 and C5.6-8 reduce to:

$$K_y = \frac{8Gr}{2-v} \qquad \text{(C5.6-10)}$$

and

$$K_\theta = \frac{8Gr^3}{3(1-v)} \qquad \text{(C5.6-11)}$$

Studies of the interaction effects in structure-soil systems have shown that, within the ranges of parameters of interest for structures subjected to earthquakes, the results are insensitive to the period-dependency of α_y and that it is sufficiently accurate for practical purposes to use the static stiffness K_y, defined by Eq. C5.6-10. However, the dynamic modifier for rocking α_θ can significantly affect the response of building structures. In the absence of more detailed analyses, for ordinary building structures with an embedment ratio $d/r < 0.5$, the factor α_θ can be estimated as follows:

R/v_sT	α_θ
<0.05	1.0
0.15	0.85
0.35	0.7
0.5	0.6

where d equals depth of embedment and r can be taken as r_m defined in Eq. 5.6-8.

The above values were derived from the solution for α_θ by Veletsos and Verbic (1973). In this solution α_θ is a function of $\tilde{T}$. To relate α_θ to T, a correction for period lengthening $(\tilde{T}/T)$ was made assuming $\bar{h}/r \sim 0.5 \text{ to } 1.0$ and Poisson's ratio $v = 0.4$.

Foundation embedment has the effect of increasing the stiffnesses K_y and K_θ. For embedded foundations for which there is positive contact between the side walls and the surrounding soil, K_y and K_θ may be determined from the following approximate formulas:

$$K_y = \left[\frac{8Gr}{2-v}\right]\left[1+\left(\frac{2}{3}\right)\left(\frac{d}{r}\right)\right] \qquad \text{(C5.6-12)}$$

and

$$K_\theta = \left[\frac{8Gr^3\alpha_\theta}{3(1-v)}\right]\left[1+2\left(\frac{d}{r}\right)\right] \quad \text{(C5.6-13)}$$

in which d is the depth of embedment. These formulas are based on finite element solutions (Kausel, 1974).

Both analyses and available test data (Erden, 1974) indicate that the effects of foundation embedment are sensitive to the condition of the backfill and that judgment must be exercised in using Eq. C5.6-12 and C5.6-13. For example, if a structure is embedded in such a way that there is no positive contact between the soil and the walls of the structure, or when any existing contact cannot reasonably be expected to remain effective during the stipulated design ground motion, stiffnesses K_y and K_θ should be determined from the formulas for surface-supported foundations. More generally, the quantity d in Eq. C5.6-12 and C5.6-13 should be interpreted as the effective depth of foundation embedment for the conditions that would prevail during the design earthquake.

The formulas for K_y and K_θ presented above are strictly valid only for foundations supported on reasonably uniform soil deposits. When the foundation rests on a surface stratum of soil underlain by a stiffer deposit with a shear wave velocity (v_s) more than twice that of the surface layer (Wallace et al., 1999), K_y and K_θ may be determined from the following two generalized formulas in which G is the shear modulus of the soft soil and D_s is the total depth of the stratum. First, using Eq. C5.6-12:

$$K_y = \left[\frac{8Gr}{2-v}\right]\left[1+\left(\frac{2}{3}\right)\left(\frac{d}{r}\right)\right]\left[1+\left(\frac{1}{2}\right)\left(\frac{r}{D_s}\right)\right]\left[1+\left(\frac{5}{4}\right)\left(\frac{d}{D_s}\right)\right] \quad \text{(C5.6-14)}$$

Second, using Eq. C5.6-13:

$$K_\theta = \left[\frac{8Gr^3\alpha_\theta}{3(1-v)}\right]\left[1+2\left(\frac{d}{r}\right)\right]\left[1+\left(\frac{1}{6}\right)\left(\frac{r}{D_s}\right)\right]\left[1+0.7\left(\frac{d}{D_s}\right)\right] \quad \text{(C5.6-15)}$$

These formulas are based on analyses of a stratum supported on a rigid base (Elsabee et al., 1977; Kausel and Roesset, 1975) and apply for $r/D_s < 0.5$ and $d/r < 1$.

The information for circular foundations presented above may be applied to mat foundations of arbitrary shapes provided the following changes are made:

1. The radius r in the expressions for K_y is replaced by r_a (Eq. 5.6-7), which represents the radius of a disk that has the area, A_o, of the actual foundation.
2. The radius r in the expressions for K_θ is replaced by r_m (Eq. 5.6-8), which represents the radius of a disk that has the moment of inertia, I_o, of the actual foundation.

For footing foundations, stiffnesses K_y and K_θ are computed by summing the contributions of the individual footings. If it is assumed that the foundation behaves as a rigid body and that the individual footings are widely spaced so that they act as independent units, the following formulas are obtained:

$$K_y = \Sigma k_{yi} \quad \text{(C5.6-16)}$$

and

$$K_\theta = \Sigma k_{xi} y_i^2 + \Sigma k_{\theta i} \quad \text{(C5.6-17)}$$

The quantity k_{yi} represents the horizontal stiffness of the i^{th} footing; k_{xi} and $k_{\theta i}$ represent, respectively, the corresponding vertical and rocking stiffnesses; and y_i represents the normal distance from the centroid of the i^{th} footing to the rocking axis of the foundation. The summations are considered to extend over all footings. The contribution to K_θ of the rocking stiffnesses of the individual footings, $k_{\theta i}$, generally is small and may be neglected.

The stiffnesses k_{yi}, k_{xi}, and $k_{\theta i}$ are defined by the formulas:

$$k_{yi} = \left(\frac{8G_i r_{ai}}{2-v}\right)\left(1+\frac{2}{3}\frac{d}{r}\right) \tag{C5.6-18}$$

$$k_{yi} = \left(\frac{4G_i r_{ai}}{1-v}\right)\left(1+0.4\frac{d}{r}\right) \tag{C5.6-19}$$

and

$$k_{\theta i} = \left(\frac{8G_i r^3{}_{mi}}{3(1-v)}\right)\left(1+2\frac{d}{r}\right) \tag{C5.6-20}$$

in which d_i is the depth of effective embedment for the i^{th} footing; G_i is the shear modulus of the soil beneath the i^{th} footing; $r_{ai} = \sqrt{A_{oi}/\pi}$ is the radius of a circular footing that has the area of the i^{th} footing, A_{oi}; and r_{mi} equals $\sqrt[4]{4I_{oi}/\pi}$ the radius of a circular footing, the moment of inertia of which about a horizontal centroidal axis is equal to that of the i^{th} footing, I_{oi}, in the direction in which the response is being evaluated.

For surface-supported footings and for embedded footings for which the side wall contact with the soil cannot be considered to be effective during the stipulated design ground motion, d_i in these formulas should be taken as zero. Furthermore, the values of G_i should be consistent with the stress levels expected under the footings and should be evaluated with due regard for the effects of the dead loads involved. This matter is considered further in subsequent sections. For closely spaced footings, consideration of the coupling effects among footings will reduce the computed value of the overall foundation stiffness. This reduction, in turn, will increase the fundamental natural period of the system, $\tilde{T}$, and increase the value of ΔV, the amount by which the base shear is reduced due to soil-structure interaction. It follows that the use of Eq. C5.6-16 and 5.6-17 will err on the conservative side in this case. The degree of conservatism involved, however, will partly be compensated by the presence of a basement slab that, even when it is not tied to the structural frame, will increase the overall stiffness of the foundation.

The values of K_y and K_θ for pile foundations can be computed in a manner analogous to that described in the preceding section by evaluating the horizontal, vertical, and rocking stiffnesses of the individual piles, k_{yi}, k_{xi}, and $k_{\theta i}$, and by combining these stiffnesses in accordance with Eq. C5.6-16 and C5.6-17.

The individual pile stiffnesses may be determined from field tests or analytically by treating each pile as a beam on an elastic subgrade. Numerous formulas are available in the literature (Tomlinson, 1994) that express these stiffnesses in terms of the modulus of the subgrade reaction and the properties of the pile itself. These stiffnesses sometimes are expressed in terms of the stiffness of an equivalent freestanding cantilever, the physical properties and cross-sectional dimensions of which are the same as those of the actual pile but the length of which is adjusted appropriately. The effective lengths of the equivalent cantilevers for horizontal motion and for rocking or bending motion are slightly different but are often assumed to be equal. On the other hand, the effective length in vertical motion is generally considerably greater.

The soil properties of interest are the shear modulus, G, or the associated shear wave velocity, v_s; the unit weight, γ; and Poisson's ratio, v. These quantities are likely to vary from point to point of a construction site, and it is necessary to use average values for the soil region that is affected by the forces acting on the foundation. The depth of significant influence is a function of the dimensions of the foundation base and of the direction of the motion involved. The effective depth may be considered to extend to about $0.75r_a$ below the foundation base for horizontal motions, $2r_a$ for vertical motions, and to about $0.75r_m$ for rocking motion. For mat foundations, the effective depth is related to the total plan dimensions of the mat whereas for structures supported on widely spaced spread footings, it is related to the dimensions of the individual footings. For closely spaced footings, the effective depth may be determined by superposition of the "pressure bulbs" induced by the forces acting on the individual footings.

Since the stress-strain relations for soils are nonlinear, the values of G and v_s also are functions of the strain levels involved. In the formulas presented above, G should be interpreted as the secant shear modulus corresponding to the significant strain level in the affected region of the foundation soil. The approximate relationship of this modulus to the modulus G_o corresponding to small amplitude strains (of the order of 10^{-3} percent or less) is given in Table 5.6-1. The backgrounds of this relationship and of the corresponding relationship for v_s/v_{so} are identified below.

The low amplitude value of the shear modulus, G_o, can most conveniently be determined from the associated value of the shear wave velocity, v_{so}, by use of Eq. C5.6-9. The latter value may be determined approximately from empirical relations or more accurately by means of field tests or laboratory tests.

The quantities G_o and v_{so} depend on a large number of factors (Hardin, 1978), the most important of which are the void ratio, e, and the average confining pressure, $\bar{\sigma}_o$. The value of the latter pressure at a given depth beneath a particular foundation may be expressed as the sum of two terms as follows:

$$\bar{\sigma}_o = \bar{\sigma}_{os} + \bar{\sigma}_{ob} \tag{C5.6-21}$$

in which $\bar{\sigma}_{os}$ represents the contribution of the weight of the soil and $\bar{\sigma}_{ob}$ represents the contribution of the superimposed weight of the structure and foundation. The first term is defined by the formula:

$$\bar{\sigma}_{os} = \left(\frac{1+2K_o}{3}\right)\gamma' x \tag{C5.6-22}$$

in which x is the depth of the soil below the ground surface, γ' is the average effective unit weight of the soil to the depth under consideration, and K_o is the coefficient of horizontal earth pressure at rest. For sands and gravel, K_o has a value of 0.5 to 0.6 whereas for soft clays, $K_o \simeq 1.0$. The pressures $\bar{\sigma}_{ob}$ developed by the weight of the structure can be estimated from the theory of elasticity (Poulos and Davis, 1974). In contrast to $\bar{\sigma}_{os}$ which increases linearly with depth, the pressures $\bar{\sigma}_{ob}$ decrease with depth. As already noted, the value of v_{so} should correspond to the average value of $\bar{\sigma}_o$ in the region of the soil that is affected by the forces acting on the foundation.

For clean sands and gravels having $e < 0.80$, the low-amplitude shear wave velocity can be calculated approximately from the formula:

$$v_{so} = c_1(2.17 - e)(\bar{\sigma})^{0.25} \tag{C5.6-23}$$

in which c_1 equals 78.2 when $\bar{\sigma}$ is in lb/ft^2 and v_{so} is in ft/sec; c_1 equals 160.4 when $\bar{\sigma}$ is in kg/cm^2 and v_{so} is in m/sec; and c_1 equals 51.0 when $\bar{\sigma}$ is in kN/m^2 and v_{so} is in m/sec.

For angular-grained cohesionless soils ($e > 0.6$), the following empirical equation may be used:

$$v_{so} = c_2(2.97 - e)(\bar{\sigma})^{0.25} \tag{C5.6-24}$$

in which c_2 equals 53.2 when $\bar{\sigma}$ is in lb/ft^2 and v_{so} is in ft/sec; c_2 equals 109.7 when $\bar{\sigma}$ is in kg/cm^2 and v_{so} is in m/sec; and c_2 equals 34.9 when $\bar{\sigma}$ is in kN/m^2 and v_{so} is in m/sec.

Equation C5.6-24 also may be used to obtain a first-order estimate of v_{so} for normally consolidated cohesive soils. A crude estimate of the shear modulus, G_o, for such soils may also be obtained from the relationship:

$$G_o = 1{,}000 s_u \tag{C5.6-25}$$

in which s_u is the shearing strength of the soil as developed in an unconfined compression test. The coefficient 1,000 represents a typical value, which varied from 250 to about 2,500 for tests on different soils (Hara et al., 1974; Hardin and Drnevich, 1975).

These empirical relations may be used to obtain preliminary, order-of-magnitude estimates. For more accurate evaluations, field measurements of v_{so} should be made. Field evaluations of the variations of v_{so}

throughout the construction site can be carried out by standard seismic refraction methods, the downhole or cross-hole methods, suspension logging, or spectral analysis with surface waves. Kramer (1996) provides an overview of these testing procedures. The disadvantage of these methods is that v_{so} is determined only for the stress conditions existing at the time of the test (usually $\bar{\sigma}_{so}$). The effect of the changes in the stress conditions caused by construction must be considered by use of Eq. C5.6-22, C5.6-23, and C5.6-24 to adjust the field measurement of v_{so} to correspond to the prototype situations. The influence of large-amplitude shearing strains may be evaluated from laboratory tests or approximated through the use of Table 5.6-1. This matter is considered further in the next two sections.

An increase in the shearing strain amplitude is associated with a reduction in the secant shear modulus, G, and the corresponding value of v_s. Extensive laboratory tests (for example, Vucetic and Dobry, 1991; Seed et al., 1984) have established the magnitudes of the reductions in v_s for both sands and clays as the shearing strain amplitude increases.

The results of such tests form the basis for the information presented in Table 5.6-1. For each severity of anticipated ground-shaking, represented by the effective peak acceleration coefficients (taken as $0.4S_{DS}$) a representative value of shearing strain amplitude was developed. A conservative value of v_s/v_{so} that is appropriate to that strain amplitude then was established. It should be emphasized that the values in Table 5.6-1 are first order approximations. More precise evaluations would require the use of material-specific shear modulus reduction curves and studies of wave propagation for the site to determine the magnitude of the soil strains induced.

It is satisfactory to assume Poisson's ratio for soils as: $v = 0.33$ for clean sands and gravels, $v = 0.40$ for stiff clays and cohesive soils, and $v = 0.45$ for soft clays. The use of an average value of $v = 0.4$ also will be adequate for practical purposes.

Regarding an alternative approach, note that Eq. 5.6-5 for the period $\tilde{T}$ of structures supported on mat foundations was deduced from Eq. 5.6-3 by making use of Eq. C5.6-10 and C5.6-11, with Poisson's ratio taken as $v = 0.4$ and with the radius r interpreted as r_a in Eq. C5.6-10 and as r_m in Eq. C5.6-11. For a nearly square foundation, for which $r_a \simeq r_m \simeq r$, Eq. 5.6-5 reduces to:

$$\tilde{T} = T\sqrt{1+25\alpha\left(\frac{r\bar{h}}{v_s^2T^2}\right)\left[1+\left(\frac{1.12\bar{h}^2}{\alpha_\theta r^2}\right)\right]} \qquad \text{(C5.6-26)}$$

The value of the relative weight parameter, α, is likely to be in the neighborhood of 0.15 for typical structures.

5.6.2.1.2 Effective damping. Equation 5.6-9 for the overall damping factor of the elastically supported structure, $\tilde{\beta}$, was determined from analyses of the harmonic response at resonance of simple systems of the type considered in Figures C5.6-2 and 5.6-3. The result is an expression of the form (Bielak, 1975; Veletsos and Nair, 1975) of:

$$\tilde{\beta} = \beta_0 + \frac{0.05}{\left(\frac{\tilde{T}}{T}\right)^3} \qquad \text{(C5.6-27)}$$

in which β_o represents the contribution of the foundation damping, considered in greater detail in the following paragraphs, and the second term represents the contribution of the structural damping. The latter damping is assumed to be of the viscous type. Equation C5.6-27 corresponds to the value of $\beta = 0.05$ used in the development of the response spectra for rigidly supported systems employed in Sec. 5.2.

The foundation damping factor, β_o, incorporates the effects of energy dissipation in the soil due to the following sources: the radiation of waves away from the foundation, known as radiation or geometric damping, and the hysteretic or inelastic action in the soil, also known as soil material damping. This factor depends on the geometry of the foundation-soil contact area and on the properties of the structure and the underlying soil deposits.

For mat foundations of circular plan that are supported at the surface of reasonably uniform soils deposits, the three most important parameters which affect the value of β_o are: the ratio $(\tilde{T}/T)$ of the fundamental natural periods of the elastically supported and the fixed-base structures, the ratio $\bar{h}/r$ of the effective height of the structure to the radius of the foundation, and the damping capacity of the soil. The latter capacity is measured by the dimensionless ratio $\Delta W_s/W_s$, in which ΔW_s is the area of the hysteresis loop in the stress-strain diagram for a soil specimen undergoing harmonic shearing deformation and W_s is the strain energy stored in a linearly elastic material subjected to the same maximum stress and strain (that is, the area of the triangle in the stress-strain diagram between the origin and the point of the maximum induced stress and strain). This ratio is a function of the magnitude of the imposed peak strain, increasing with increasing intensity of excitation or level of strain.

The variation of β_o with $\tilde{T}/T$ and $\bar{h}/r$ is given in Figure 5.6-1 for two levels of excitation. The dashed lines, which are recommended for values of the effective peak ground acceleration (taken as $0.4S_{DS}$) equal to or less than 0.10, correspond to a value of $\Delta W_s/W_s \simeq 0.3$, whereas the solid lines, which are recommended for values of effective peak ground acceleration equal to or greater than 0.20, correspond to a value of $\Delta W_s/W_s \simeq 1$. These curves are based on the results of extensive parametric studies (Veletsos, 1977; Veletsos and Meek, 1974; Veletsos and Nair, 1975) and represent average values. For the ranges of parameters that are of interest in practice, however, the dispersion of the results is small.

For mat foundations of arbitrary shape, the quantity r in Figure 5.6-1 should be interpreted as a characteristic length that is related to the length of the foundation, L_o, in the direction in which the structure is being analyzed. For short, squatty structures for which $\bar{h}/L_o \le 0.5,$ the overall damping of the structure-foundation system is dominated by the translational action of the foundation, and it is reasonable to interpret r as r_a, the radius of a disk that has the same area as that of the actual foundation (see Eq. 5.6-7). On the other hand, for structures with $\bar{h}/L_o \ge 0.1,$ the interaction effects are dominated by the rocking motion of the foundation, and it is reasonable to define r as the radius r_m of a disk whose static moment of inertia about a horizontal centroidal axis is the same as that of the actual foundation normal to the direction in which the structure is being analyzed (see Eq. 5.6-8).

Subject to the qualifications noted in the following section, the curves in Figure 5.6-1 also may be used for embedded mat foundations and for foundations involving spread footings or piles. In the latter cases, the quantities A_o and I_o in the expressions for the characteristic foundation length, r, should be interpreted as the area and the moment of inertia of the load-carrying foundation.

In the evaluation of the overall damping of the structure-foundation system, no distinction has been made between surface-supported foundations and embedded foundations. Since the effect of embedment is to increase the damping capacity of the foundation (Bielak, 1975; Novak, 1974; Novak and Beredugo, 1972) and since such an increase is associated with a reduction in the magnitude of the forces induced in the structure, the use of the recommended requirements for embedded structures will err on the conservative side.

There is one additional source of conservatism in the application of the recommended requirements to structures with embedded foundations. It results from the assumption that the free-field ground motion at the foundation level is independent of the depth of foundation embedment. Actually, there is evidence to the effect that the severity of the free-field excitation decreases with depth (Seed et al., 1977). This reduction is ignored both in Sec. 5.6 and in the requirements for rigidly supported structures presented in Sec. 5.2 and 5.3.

Equations 5.6-9 and C5.6-28, in combination with the information presented in Figure 5.6-1, may lead to damping factors for the structure-soil system, $\tilde{\beta},$ that are smaller than the structural damping factor, β. However, since the representative value of $\beta = 0.05$ used in the development of the design requirements for rigidly supported structures is based on the results of tests on actual structures, it reflects the damping of the full structure-soil system, not merely of the component contributed by the superstructure. Thus, the value of $\tilde{\beta}$ determined from Eq. 5.6-9 should never be taken less than β, and a minimum value of $\tilde{\beta} = \beta = 0.05$ has been imposed. The use of values of $\tilde{\beta} > \beta$ is justified by the fact that the experimental values correspond to extremely small amplitude motions and do not reflect the effects of the higher soil damping capacities corresponding to the large soil strain levels associated with the design ground motions. The effects of the higher soil damping capacities are appropriately reflected in the values of β_o presented in Figure 5.6-1.

There are, however, some exceptions. For foundations involving a soft soil stratum of reasonably uniform properties underlain by a much stiffer, rock-like material with an abrupt increase in stiffness, the radiation damping effects are practically negligible when the natural period of vibration of the stratum in shear,

$$T_s = \frac{4D_s}{v_s} \tag{C5.6-28}$$

is smaller than the natural period of the flexibly supported structure, $\tilde{T}$. The quantity D_s in this formula represents the depth of the stratum. It follows that the values of β_o presented in Figure 5.6-1 are applicable only when:

$$\frac{T_s}{\tilde{T}} = \frac{4D_s}{v_s\tilde{T}} \geq 1 \tag{C5.6-29}$$

For

$$\frac{T_s}{\tilde{T}} = \frac{4D_s}{v_s\tilde{T}} < 1 \tag{C5.6-30}$$

the effective value of the foundation damping factor, β'_o, is less than β_o, and it is approximated by the second degree parabola defined by Eq. 5.6-10.

For $T_s/\tilde{T} = 1$, Eq. 5.6-10 leads to $\beta'_o = \beta_o$ whereas for $T_s/\tilde{T} = 0$, it leads to $\beta'_0 = 0$, a value that clearly does not provide for the effects of material soil damping. It may be expected, therefore, that the computed values of β'_0 corresponding to small values of $T_s/\tilde{T}$ will be conservative. The conservatism involved, however, is partly compensated by the requirement that $\tilde{\beta}$ be no less than $\tilde{\beta} = \beta = 0.05$.

5.6.2.2 and 5.6.2.3 Vertical distribution of seismic forces and other effects. The vertical distributions of the equivalent lateral forces for flexibly and rigidly supported structures are generally different. However, the differences are inconsequential for practical purposes, and it is recommended that the same distribution be used in both cases, changing only the magnitude of the forces to correspond to the appropriate base shear. A greater degree of refinement in this step would be inconsistent with the approximations embodied in the requirements for rigidly supported structures.

With the vertical distribution of the lateral forces established, the overturning moments and the torsional effects about a vertical axis are computed as for rigidly supported structures. The above procedure is applicable to planar structures and, with some extension, to three-dimensional structures.

5.6.3 Response spectrum procedure. Studies of the dynamic response of elastically supported, multi-degree-of-freedom systems (Bielak, 1976; Chopra and Gutierrez, 1974; Veletsos, 1977) reveal that, within the ranges of parameters that are of interest in the design of structures subjected to earthquakes, soil-structure interaction affects substantially only the response component contributed by the fundamental mode of vibration of the superstructure. In this section, the interaction effects are considered only in evaluating the contribution of the fundamental structural mode. The contributions of the higher modes are computed as if the structure were fixed at the base, and the maximum value of a response quantity is determined, as for rigidly supported structures, by taking the square root of the sum of the squares of the maximum modal contributions.

The interaction effects associated with the response in the fundamental structural mode are determined in a manner analogous to that used in the equivalent lateral force procedure, except that the effective weight and effective height of the structure are computed so as to correspond exactly to those of the fundamental natural mode of the fixed-base structure. More specifically, $\bar{W}$ is computed from:

$$\bar{W} = \bar{W}_1 = \frac{\left(\Sigma w_i \phi_{i1}\right)^2}{\Sigma w_i \phi_{i1}^2} \qquad \text{(C5.6-31)}$$

which is the same as Eq. 5.3-2, and $\bar{h}$ is computed from Eq. 5.6-13. The quantity ϕ_{i1} in these formulas represents the displacement amplitude of the i^{th} floor level when the structure is vibrating in its fixed-base, fundamental natural mode. The structural stiffness, $\bar{k}$, is obtained from Eq. 5.6-4 by taking $\bar{W} = \bar{W}_1$ and using for T the fundamental natural period of the fixed-base structure, T_1. The fundamental natural period of the interacting system, $\tilde{T}_1$, is then computed from Eq. 5.6-3 (or Eq. 5.6-5 when applicable) by taking $T = T_1$. The effective damping in the first mode, β, is determined from Eq. 5.6-9 (and Eq. 5.6-10 when applicable) in combination with the information given in Figure 5.6-1. The quantity $\bar{h}$ in the latter figure is computed from Eq. 5.6-13.

With the values of $\tilde{T}_1$ and $\tilde{\beta}_1$ established, the reduction in the base shear for the first mode, ΔV_1, is computed from Eq. 5.6-2. The quantities C_s and $\tilde{C}_s$ in this formula should be interpreted as the seismic coefficients corresponding to the periods T_1 and $\tilde{T}_1$, respectively; $\tilde{\beta}$ should be taken equal to $\tilde{\beta}_1$; and $\bar{W}$ should be determined from Eq. C5.6-31.

The sections on lateral forces, shears, overturning moments, and displacements follow directly from what has already been noted in this and the preceding sections and need no elaboration. It may only be pointed out that the first term within the brackets on the right side of Eq. 5.6-14 represents the contribution of the foundation rotation.

5.6.3.3 Design values. The design values of the modified shears, moments, deflections, and story drifts should be determined as for structures without interaction by taking the square root of the sum of the squares of the respective modal contributions. In the design of the foundation, the overturning moment at the foundation-soil interface determined in this manner may be reduced by 10 percent as for structures without interaction.

The effects of torsion about a vertical axis should be evaluated in accordance with the requirements of Sec. 5.2.4 and the *P*-delta effects should be evaluated in accordance with the requirements of Sec. 5.2.6.2, using the story shears and drifts determined in Sec. 5.6.3.2.

Other methods of considering the effects of soil-structure interaction. The procedures proposed in the preceding sections for incorporating the effects of soil-structure interaction provide sufficient flexibility and accuracy for practical applications. Only for unusual structures and only when the requirements indicate that the interaction effects are of definite consequence in design, would the use of more elaborate procedures be justified. Some of the possible refinements, listed in order of more or less increasing complexity, are:

1. Improve the estimates of the static stiffnesses of the foundation, K_y and K_θ, and of the foundation damping factor, β_0, by considering in a more precise manner the foundation type involved, the effects of foundation embedment, variations of soil properties with depth, and hysteretic action in the soil. Solutions may be obtained in some cases with analytical or semi-analytical formulations and in others by application of finite difference or finite element techniques. A concise review of available analytical formulations is provided in Gazetas (1991). It should be noted, however, that these

solutions involve approximations of their own that may offset, at least in part, the apparent increase in accuracy.

2. Improve the estimates of the average properties of the foundation soils for the stipulated design ground motion. This would require both laboratory tests on undisturbed samples from the site and studies of wave propagation for the site. The laboratory tests are needed to establish the actual variations with shearing strain amplitude of the shear modulus and damping capacity of the soil, whereas the wave propagation studies are needed to establish realistic values for the predominant soil strains induced by the design ground motion.
3. Incorporate the effects of interaction for the higher modes of vibration of the structure, either approximately by application of the procedures recommended in Bielak (1976), Roesset et al. (1973), and Tsai (1974) or by more precise analyses of the structure-soil system. The latter analyses may be implemented either in the time domain or by application of the impulse response functions presented in Veletsos and Verbic (1974). However, the frequency domain analysis is limited to systems that respond within the elastic range while the approach involving the use of the impulse response functions is limited, at present, to soil deposits that can adequately be represented as a uniform elastic halfspace. The effects of yielding in the structure and/or supporting medium can be considered only approximately in this approach by representing the supporting medium by a series of springs and dashpots whose properties are independent of the frequency of the motion and by integrating numerically the governing equations of motion (Parmelee et al., 1969).
4. Analyze the structure-soil system by finite element method (for example, Lysmer et al., 1981; Borja et al., 1992), taking due account of the nonlinear effects in both the structure and the supporting medium.

It should be emphasized that, while these more elaborate procedures may be appropriate in special cases for design verification, they involve their own approximations and do not eliminate the uncertainties that are inherent in the modeling of the structure-foundation-soil system and in the specification of the design ground motion and of the properties of the structure and soil.

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