Structural Analysis for Performance-Based Earthquake Engineering

- Basic modeling concepts
- **Nonlinear static pushover analysis**
- Nonlinear dynamic response history analysis
- Incremental nonlinear analysis
- Probabilistic approaches

Nonlinear Static Pushover Analysis

- Why pushover analysis?
- Basic overview of method
- Details of various steps
- Discussion of assumptions
- Improved methods

Why Pushover Analysis?

- Performance-based methods require reasonable estimates of inelastic deformation or damage in structures.
- Elastic Analysis is not capable of providing this information.
- Nonlinear dynamic response history analysis is capable of providing the required information, but may be very time-consuming.

Why Pushover Analysis?

- Nonlinear static pushover analysis may provide reasonable estimates of location of inelastic behavior.

- Pushover analysis alone is not capable of providing estimates of maximum deformation. Additional analysis must be performed for this purpose. The fundamental issue is…
How Far to Push?

Why Pushover Analysis?

- It is important to recognize that the purpose of pushover analysis is not to predict the actual response of a structure to an earthquake. (It is unlikely that nonlinear dynamic analysis can predict the response.)

- The minimum requirement for any method of analysis, including pushover, is that it must be "good enough for design".

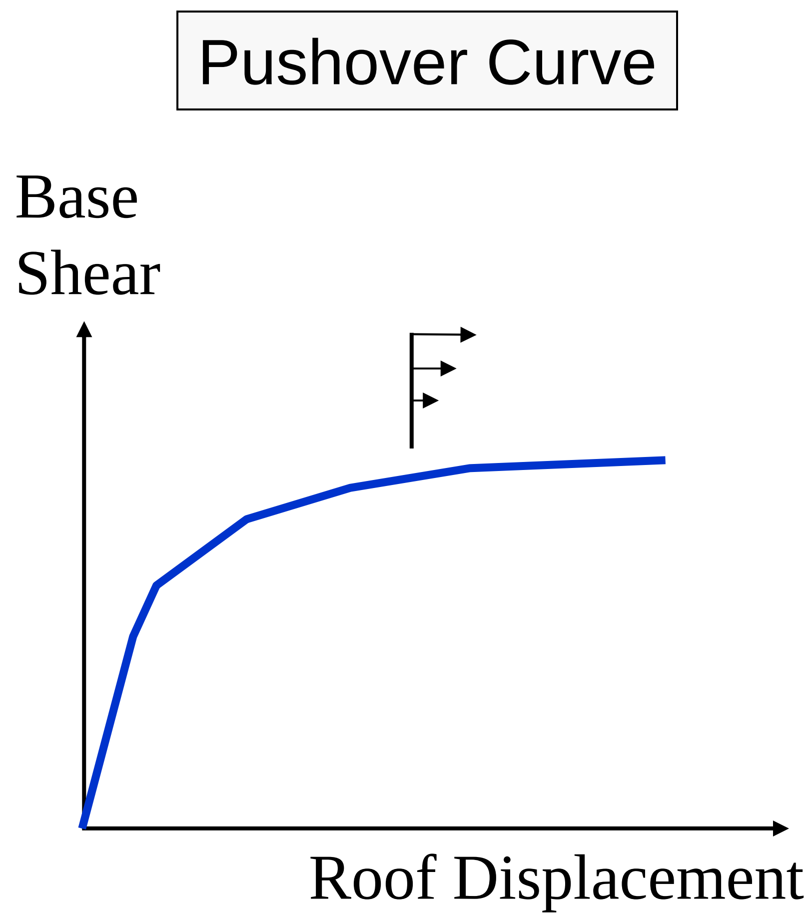
Basic Overview of Method

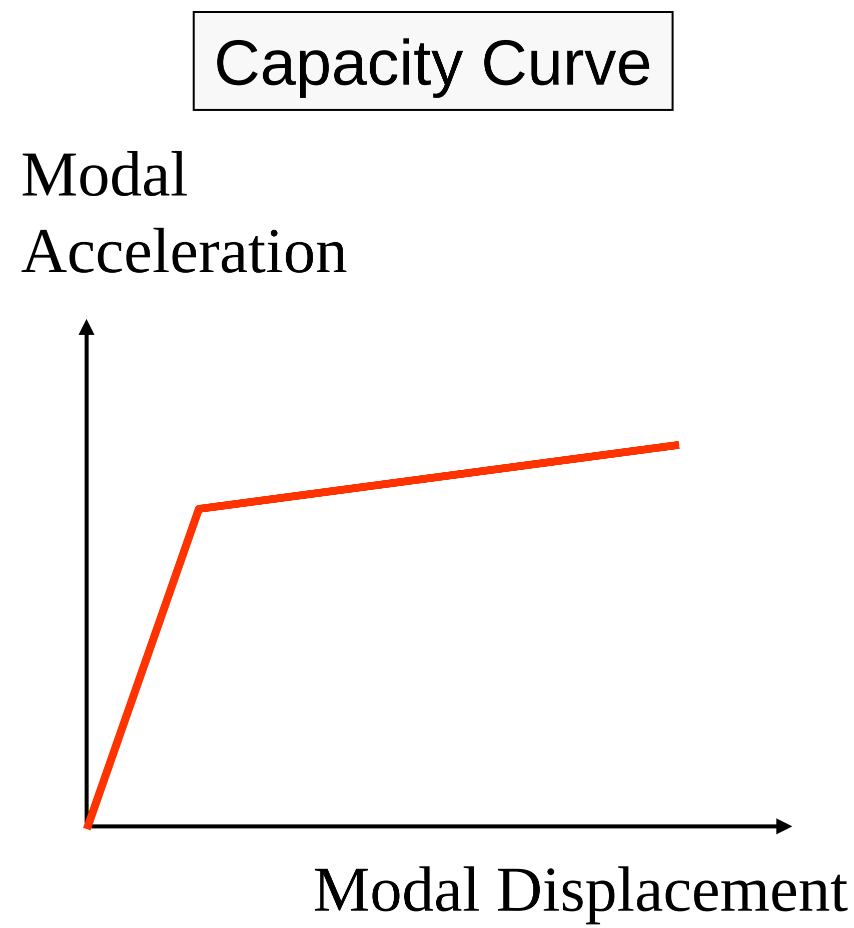
- Development of Capacity Curve
- Prediction of "Target Displacement"
  - Capacity-Spectrum Approach (ATC 40)
  - Simplified Approach (FEMA 273, NEHRP)
  - Uncoupled Modal Response History
  - Modal Pushover

Development of the Capacity Curve (ATC 40 Approach)

1. Develop Analytical Model of Structure Including:
   - Gravity loads
   - Known sources of inelastic behavior
   - P-Delta Effects
2. Compute Modal Properties:
   - Periods and Mode Shapes
   - Modal Participation Factors
   - Effective Modal Mass
3. Assume Lateral Inertial Force Distribution
4. Construct Pushover Curve
5. Transform Pushover Curve to 1st Mode Capacity Curve
6. Simplify Capacity Curve (Use bilinear approximation)

Development of the Capacity Curve

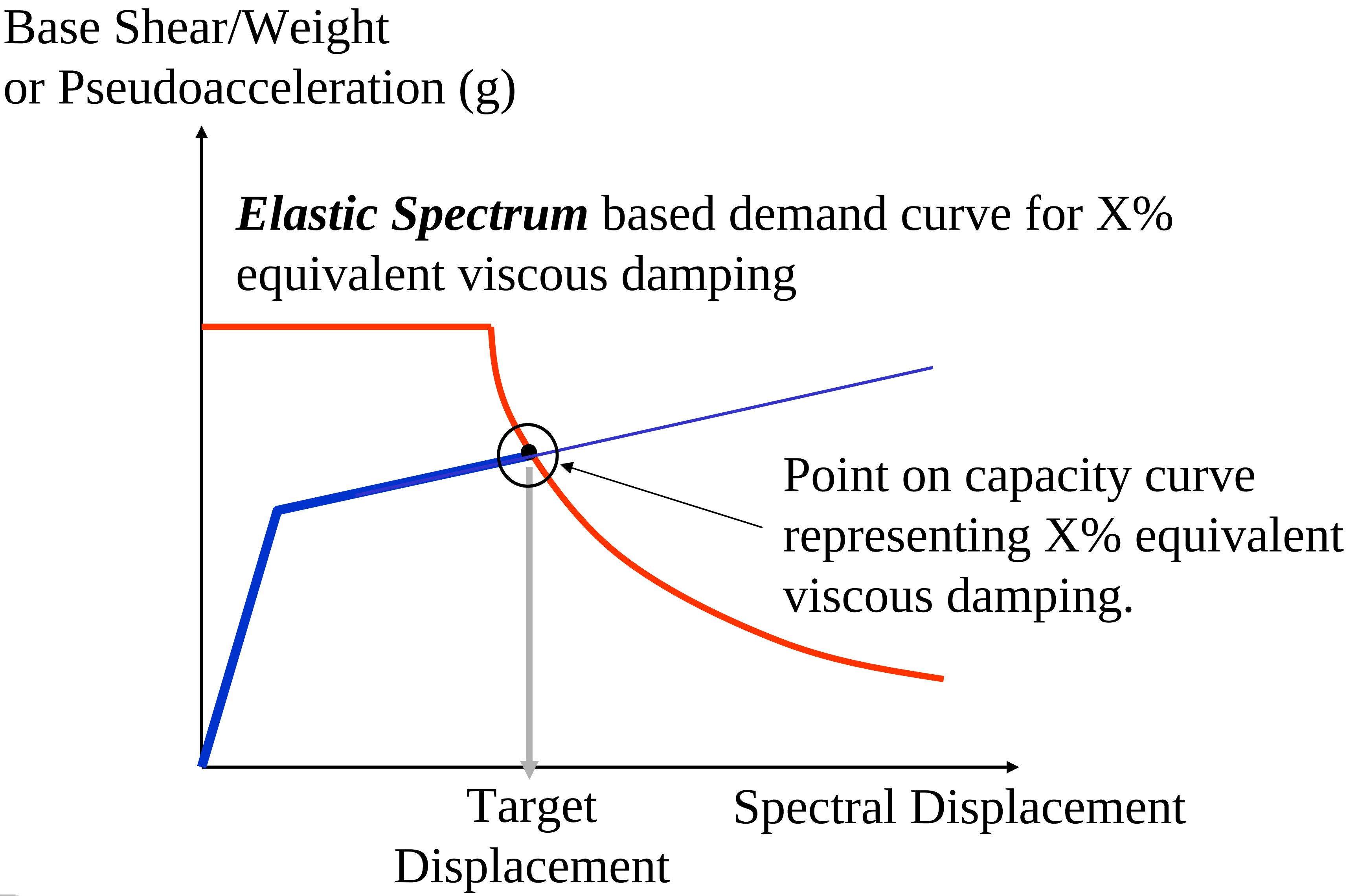




Development of the Demand Curve

1. Assume Seismic Hazard Level (e.g 2% in 50 years)
2. Develop 5% Damped *ELASTIC* Response Spectrum
3. Modify for Site Effects
4. Modify for Expected Performance and Equivalent Damping
5. Convert to Displacement-Acceleration Format

Elastic Spectrum Based Target Displacement



Review of MDOF Dynamics (1)

Original Equations of Motion:

$$M\ddot{u} + C\dot{u} + Ku = -MR\ddot{u}_g \qquad K\Phi = M\Phi\Omega^2 \qquad R = \begin{Bmatrix} 1 \\ 1 \\ \cdot \\ 1 \end{Bmatrix}$$

Transformation to Modal Coordinates:

$$u = \Phi y$$

$$\Phi = \left[\phi_1 \; \phi_2 \; \phi_3 \; \ldots \; \phi_n\right] \qquad y = \begin{Bmatrix} y_1 \\ y_2 \\ \cdot \\ y_n \end{Bmatrix}$$

$$M\Phi\ddot{y} + C\Phi\dot{y} + K\Phi y = -MR\ddot{u}_g$$

Review of MDOF Dynamics (2)

Use of Orthogonality Relationships:

$$\Phi^{\mathrm{T}} M \Phi \ddot{y} + \Phi^{\mathrm{T}} C \Phi \dot{y} + \Phi^{\mathrm{T}} K \Phi y = -\Phi^{\mathrm{T}} M R \ddot{u}_g$$

$$\Phi^{\mathrm{T}} M \Phi = M^*$$

$$\Phi^{\mathrm{T}} C \Phi = C^*$$

$$\Phi^{\mathrm{T}} K \Phi = K^*$$

$$\phi_i^{\mathrm{T}} M \phi_i = m_i^*$$

$$\phi_i^{\mathrm{T}} C \phi_i = c_i^*$$

$$\phi_i^{\mathrm{T}} K \phi_i = k_i^*$$

SDOF equation in Mode i :

$$m_i^* \ddot{y}_i + c_i^* \dot{y}_i + k_i^* y_i = -\phi_i^{\mathrm{T}} M R \ddot{u}_g$$

Review of MDOF Dynamics (3)

Simplify by dividing through by m_i^*

and noting

$$\frac{c_i^*}{m_i^*} = 2\xi_i\omega_i \qquad \frac{k_i^*}{m_i^*} = \omega_i^2$$

$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}\ddot{u}_g = -\Gamma_i\ddot{u}_g$$

Review of MDOF Dynamics (4)

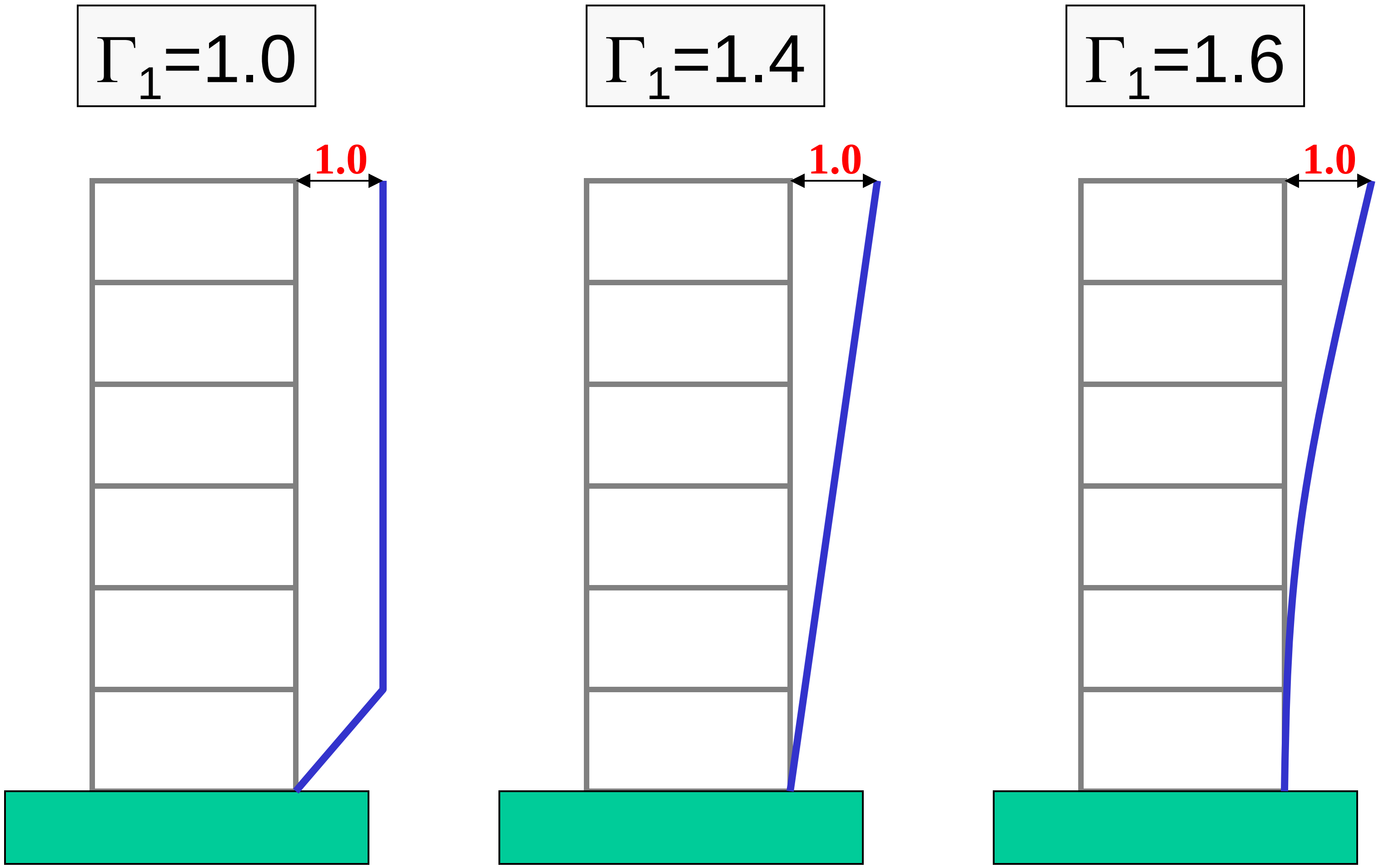
$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega^2 y_i = -\frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}\ddot{u}_g = -\Gamma_i\ddot{u}_g$$

Modal Participation Factor:

$$\Gamma_i = \frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}$$

Important Note: Γ_i depends on mode shape scaling

Variation of First Mode Participation Factor with First Mode Shape



Review of MDOF Dynamics (5)

Any Mode of MDOF system

$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^{\,2} y_i = -\Gamma_i\ddot{u}_g$$

SDOF system

$$\ddot{D}_i + 2\xi_i\omega_i\dot{D}_i + \omega_i^{\,2} D_i = -\ddot{u}_g$$

If we obtain the displacement $D_i(t)$ from the response of a SDOF we must multiply by Γ_1 to obtain the modal amplitude response $y_i(t)$. history

$$y_1(t) = \Gamma_1 D_i(t)$$

Review of MDOF Dynamics (6)

If we run a SDOF Response history analysis:

$$y_i(t) = \Gamma_i D_i(t)$$

If we use a response spectrum:

$$y_{i,\max} = \Gamma_i D_{i,\max}$$

Review of MDOF Dynamics (7)

In general

$$y_i(t) = \Gamma_i D_i(t)$$

Recalling

$$u_i(t) = \phi_i y_i(t)$$

Substituting

$$u_i(t) = \Gamma_i \phi_i D_i(t)$$

Review of MDOF Dynamics (8)

Applied “static” forces required to produce $u_i(t)$:

$$F_i(t) = Ku_i(t) = \Gamma_i K\phi_i D_i(t)$$

Recall $K\phi_i = \omega_i^2 M\phi_i$

$$F_i(t) = \Gamma_i M\phi_i\omega_i^2 D_i(t) = \Gamma_i M\phi_i a_i(t)$$

$$\boxed{F_i(t) = S_i a_i(t)}$$ where $S_i = \Gamma_i M\phi_i$

Review of MDOF Dynamics (9)

Total shear in mode:

$$V_i = F_i^T R$$

$$V_i(t) = \Gamma_i (M\phi_i)^T R a_i(t) = \Gamma_i \phi_i^T M R a_i(t)$$

$$V_i(t) = \hat{M}_i a_i(t)$$

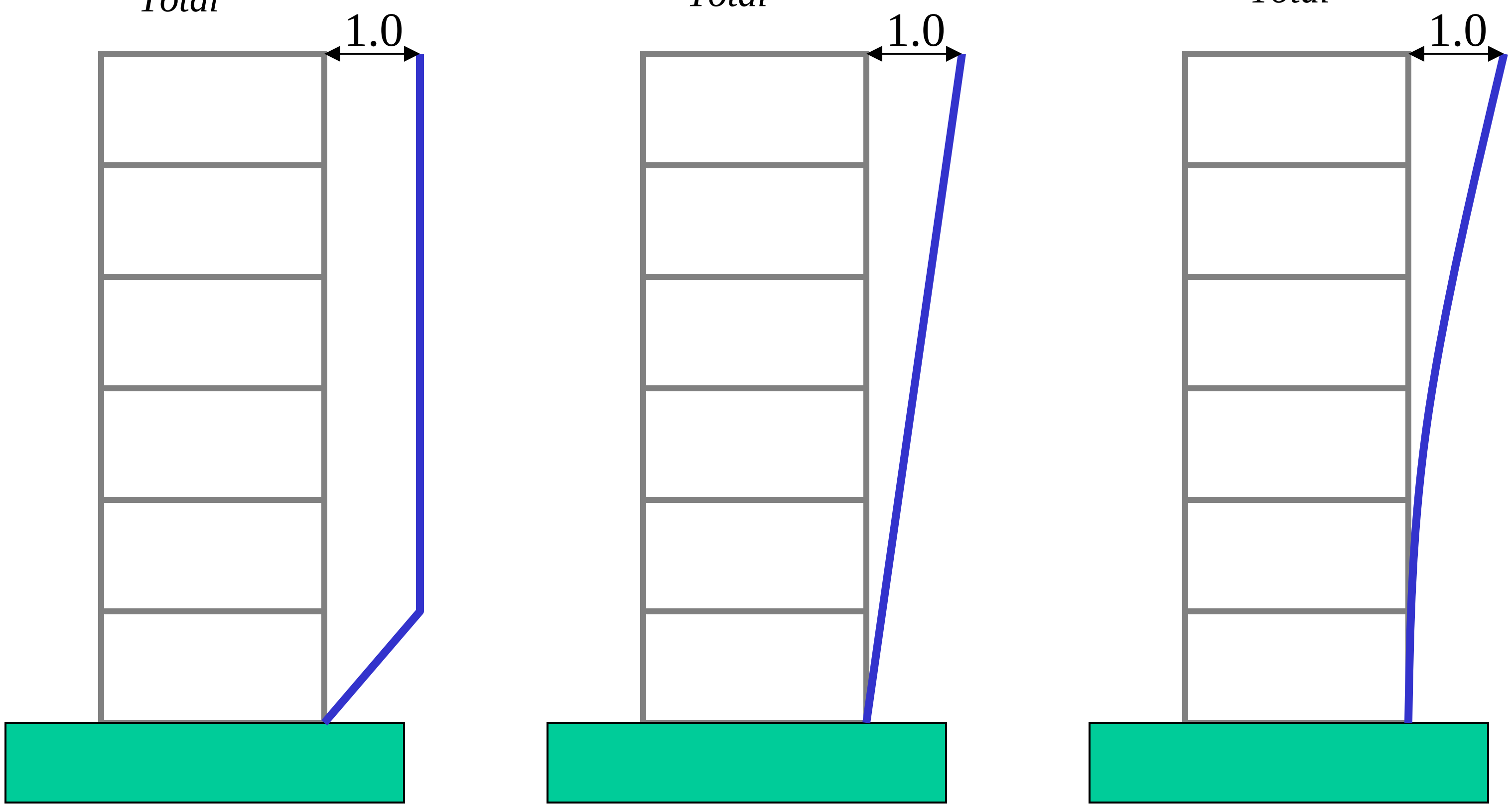
Effective Modal Mass:

$$\hat{M}_i = \frac{[\phi_i^{\mathrm{T}} M R]^2}{\phi_i^{\mathrm{T}} M \phi_i}$$

Important Note: $\hat{M}_i$ ***does NOT*** depend on mode shape scaling

Variation of First Mode Effective Mass with First Mode Shape

$$\frac{\hat{M}_1}{M_{Total}} = 1.0 \qquad \frac{\hat{M}_1}{M_{Total}} = 0.9 \qquad \frac{\hat{M}_1}{M_{Total}} = 0.8$$



Review of MDOF Dynamics (10)

$$S_1 + S_2 + ... + S_n = MR$$

$$\sum_{k=1}^{n} S_{i,k} = \hat{M}_i$$

Simple Numerical Example

$$K = \begin{bmatrix} 50 & -50 & 0 \\ -50 & 110 & -60 \\ 0 & -60 & 130 \end{bmatrix} \qquad M = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}$$

$$S_1 = \begin{Bmatrix} 1.267 \\ 1.060 \\ 0.600 \end{Bmatrix} \qquad S_2 = \begin{Bmatrix} -0.338 \\ 0.223 \\ 0.428 \end{Bmatrix} \qquad S_3 = \begin{Bmatrix} 0.071 \\ -0.183 \\ 0.172 \end{Bmatrix}$$

$$\sum_{k=1}^{3} S_{1,k} = 2.927 \qquad \sum_{k=1}^{3} S_{2,k} = 0.313 \qquad \sum_{k=1}^{3} S_{3,k} = 0.060$$

$$S_1 + S_2 + S_3 = \begin{Bmatrix} 1.0 \\ 1.1 \\ 1.2 \end{Bmatrix}$$

Review of MDOF Dynamics (11)

Displacement Response in single mode:

$$u_i(t) = \Gamma_i \phi_i D_i(t)$$

From Response-History or Response Spectrum Analysis

Total shear in single mode:

$$V_i(t) = \hat{M}_i a_i(t)$$

First Mode Response as Function of System Response

Modal Displacement:

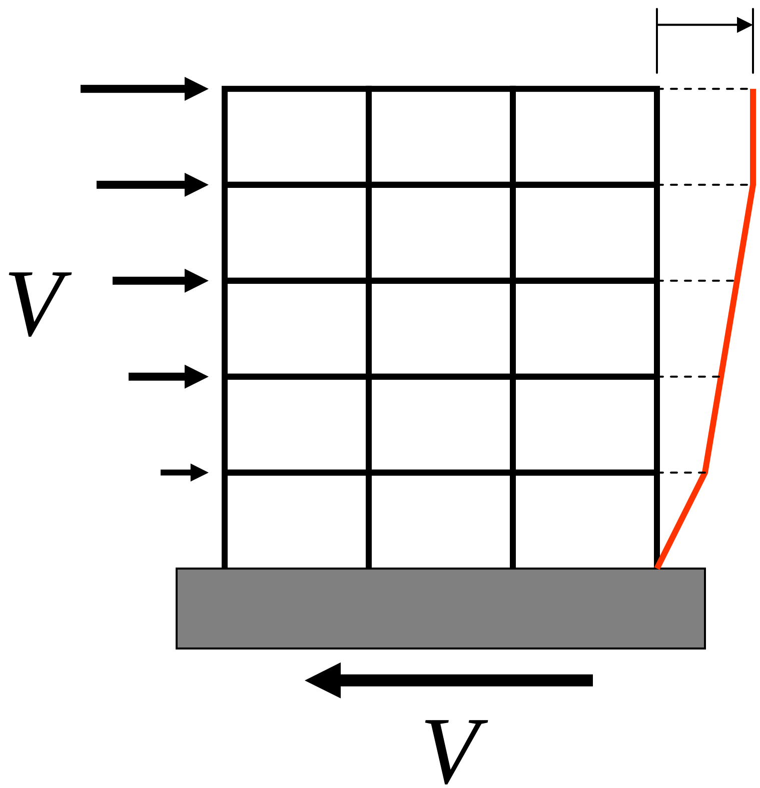
$$D_1(t) = \frac{u_{1,roof}(t)}{\Gamma_1 \phi_{1,roof}}$$

Modal Acceleration:

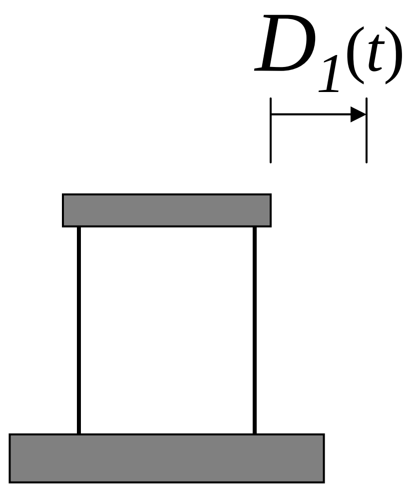
$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$

Converting Pushover Curve to Capacity Curve

$$u_{1,roof}(t) = \Gamma_1 \phi_{1,roof} D_1(t)$$

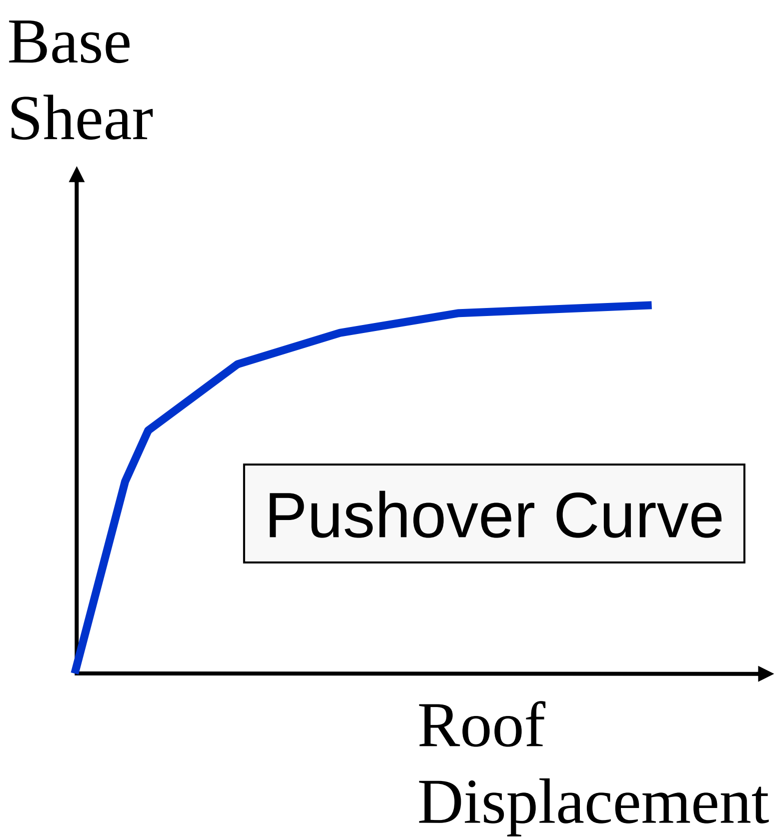


First Mode System (natural coords)



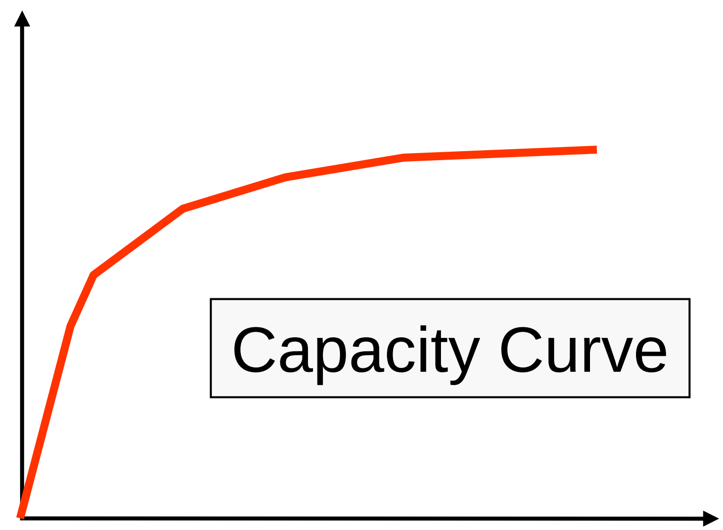
First Mode SDOF System (modal coords)

Converting Pushover Curve to Capacity Curve



Modal Acceleration

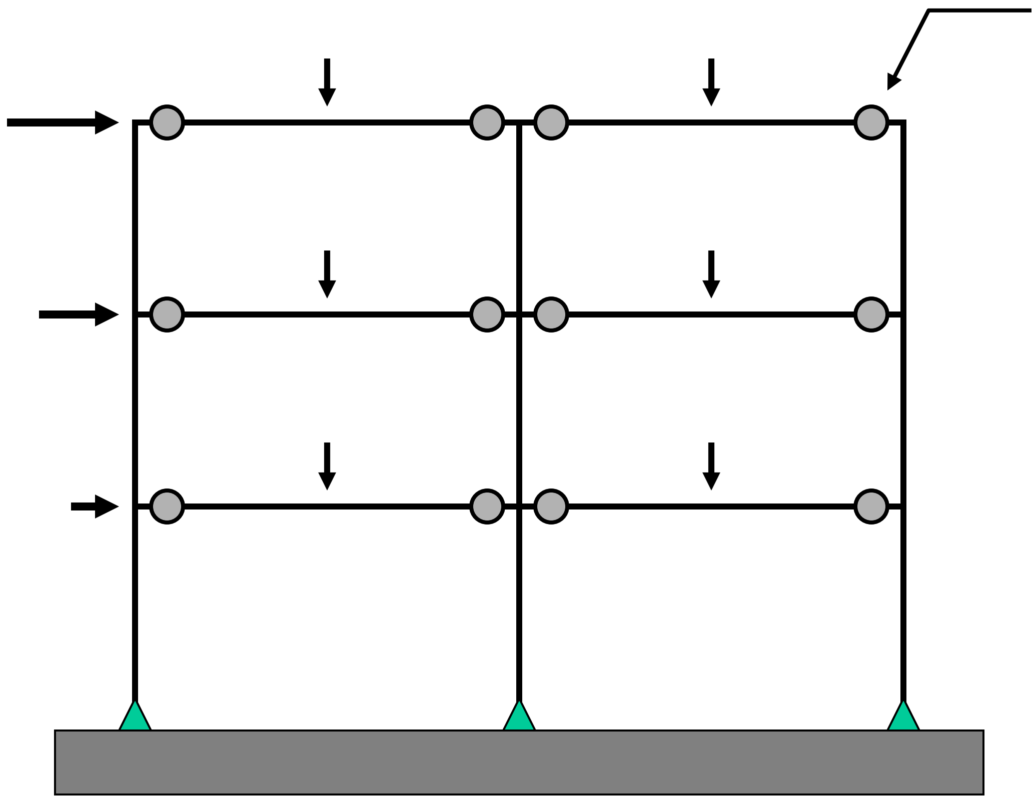
$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$



Modal Displacement

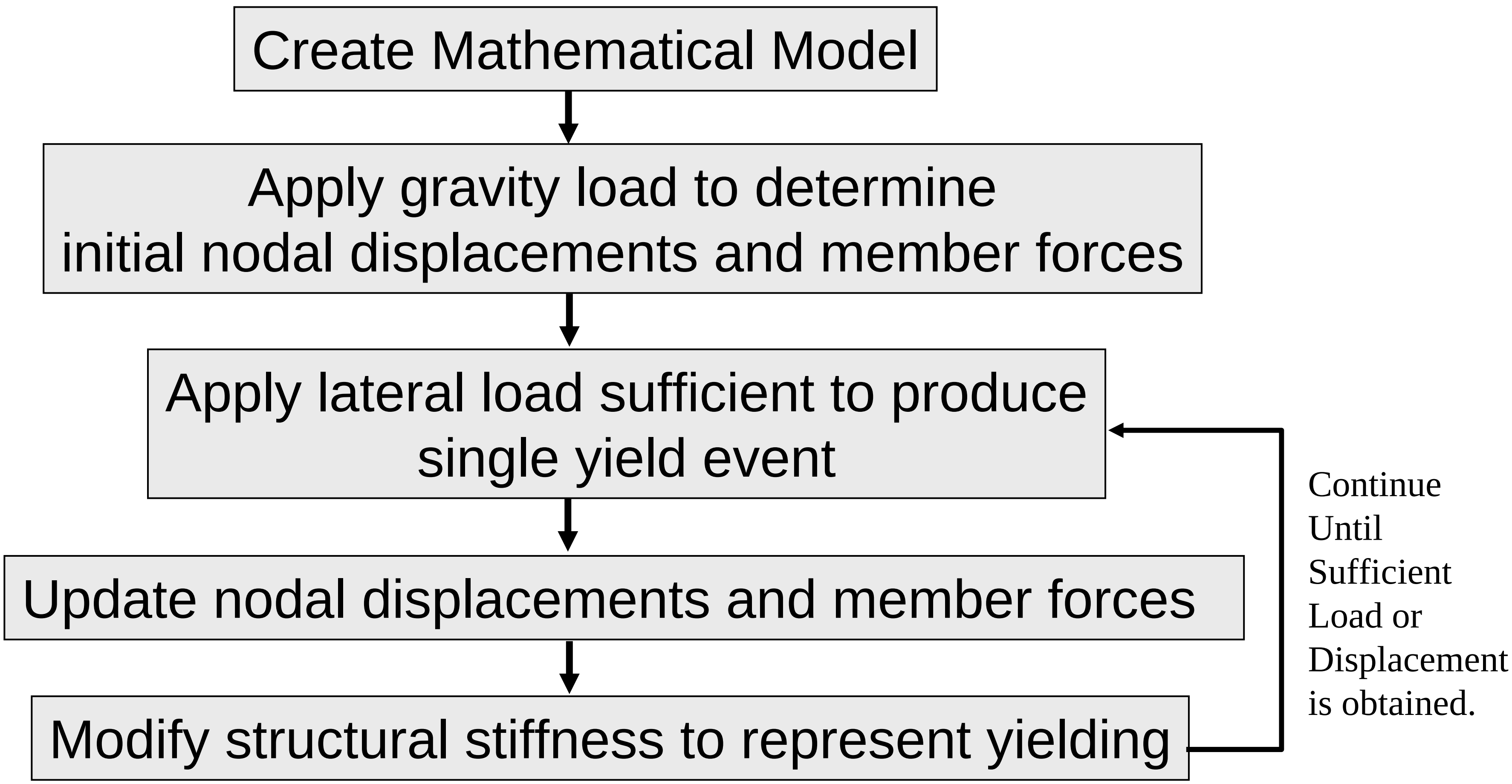
$$D_1(t) = \frac{u_1(t)}{\Gamma_1 \phi_{1,roof}}$$

Development of Pushover Curve

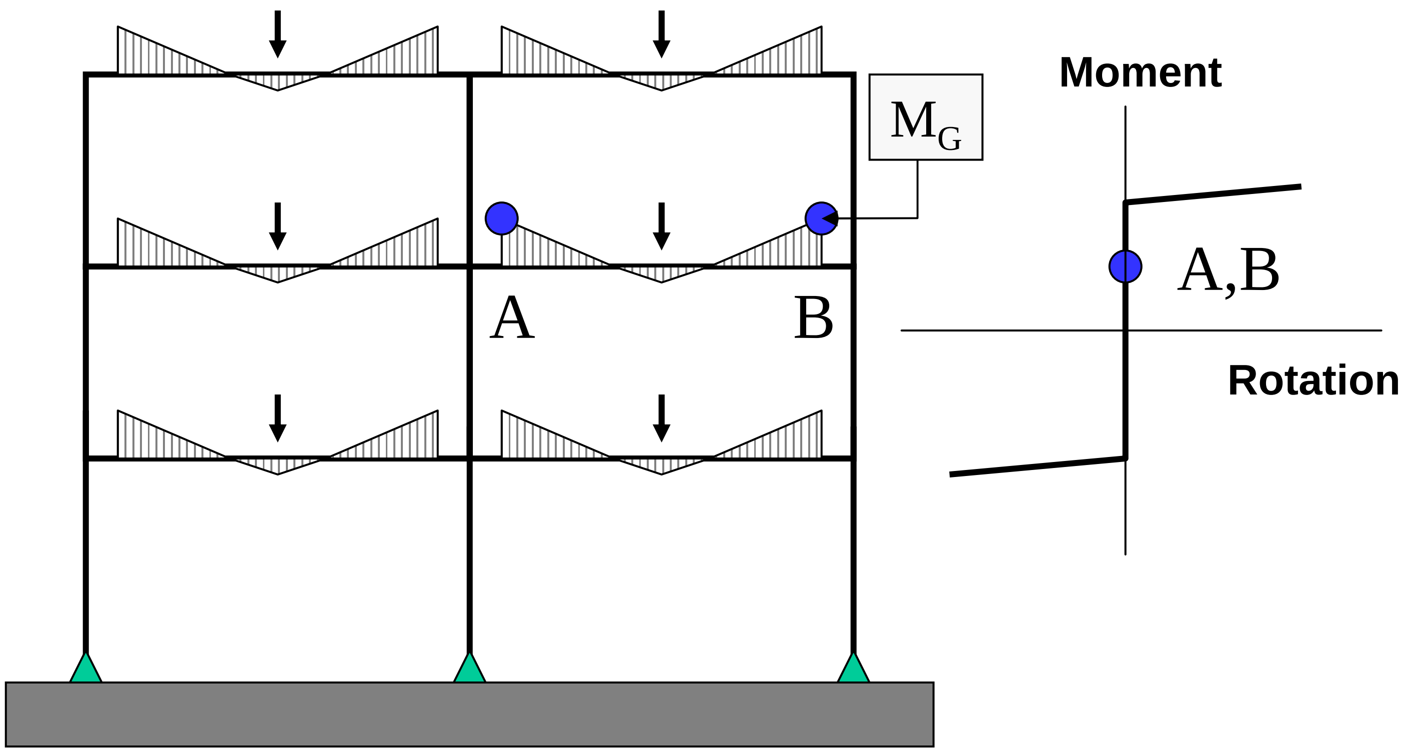


Potential Plastic Hinge Location (Must be predicted and possibly corrected)

Event-to-Event Pushover Analysis

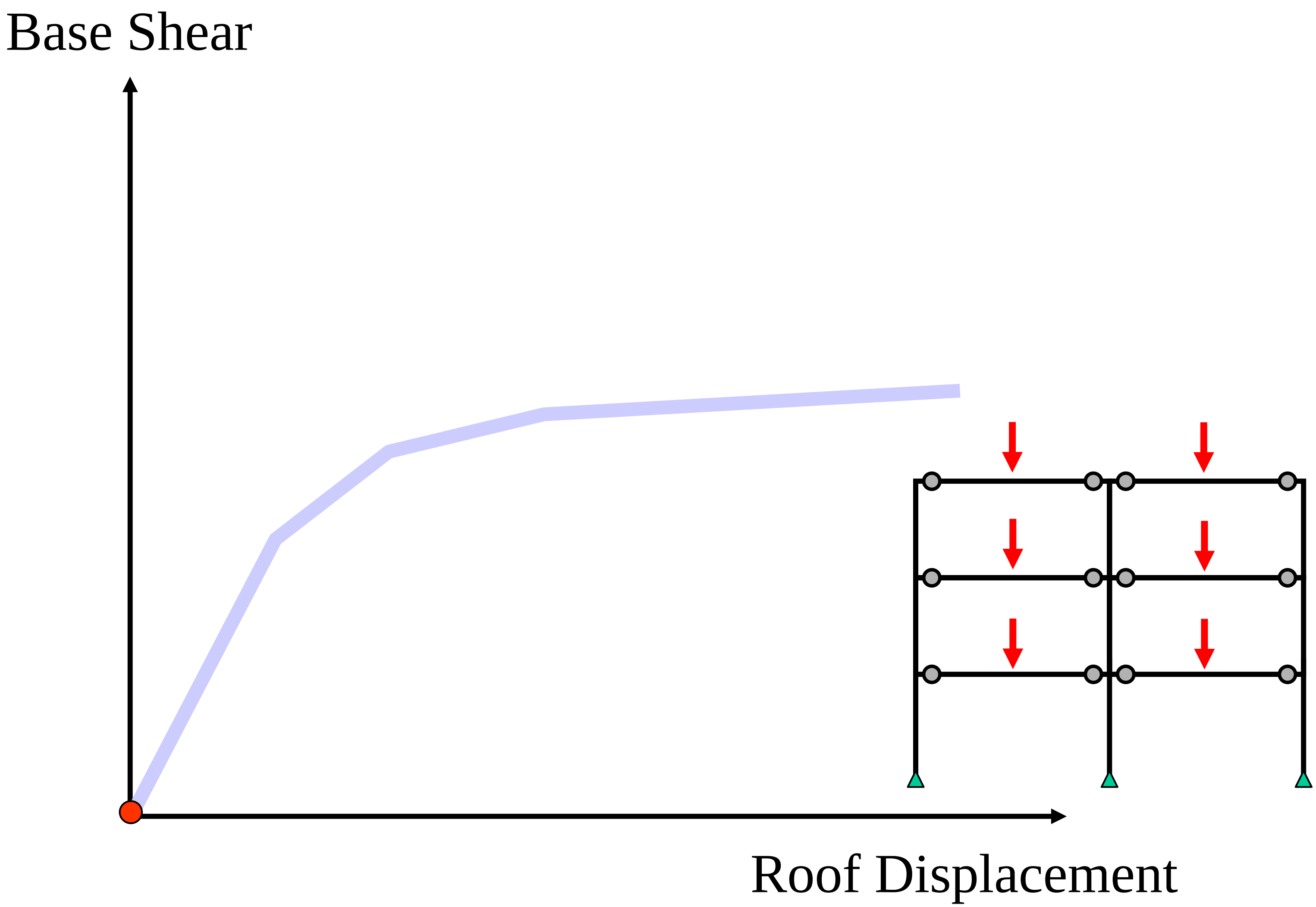


Initial Gravity Load Analysis

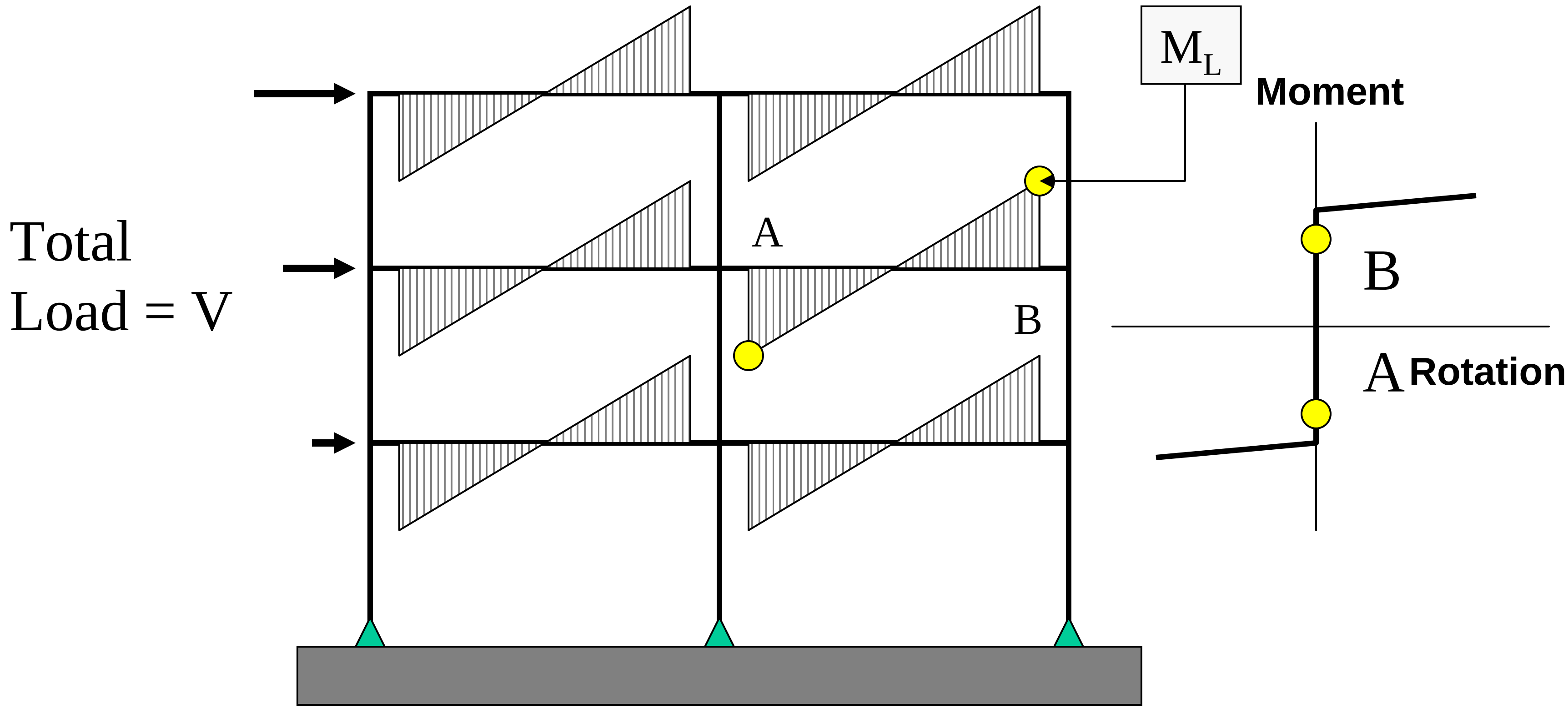


Moments plotted on tension side.

Analysis 1: Gravity Analysis



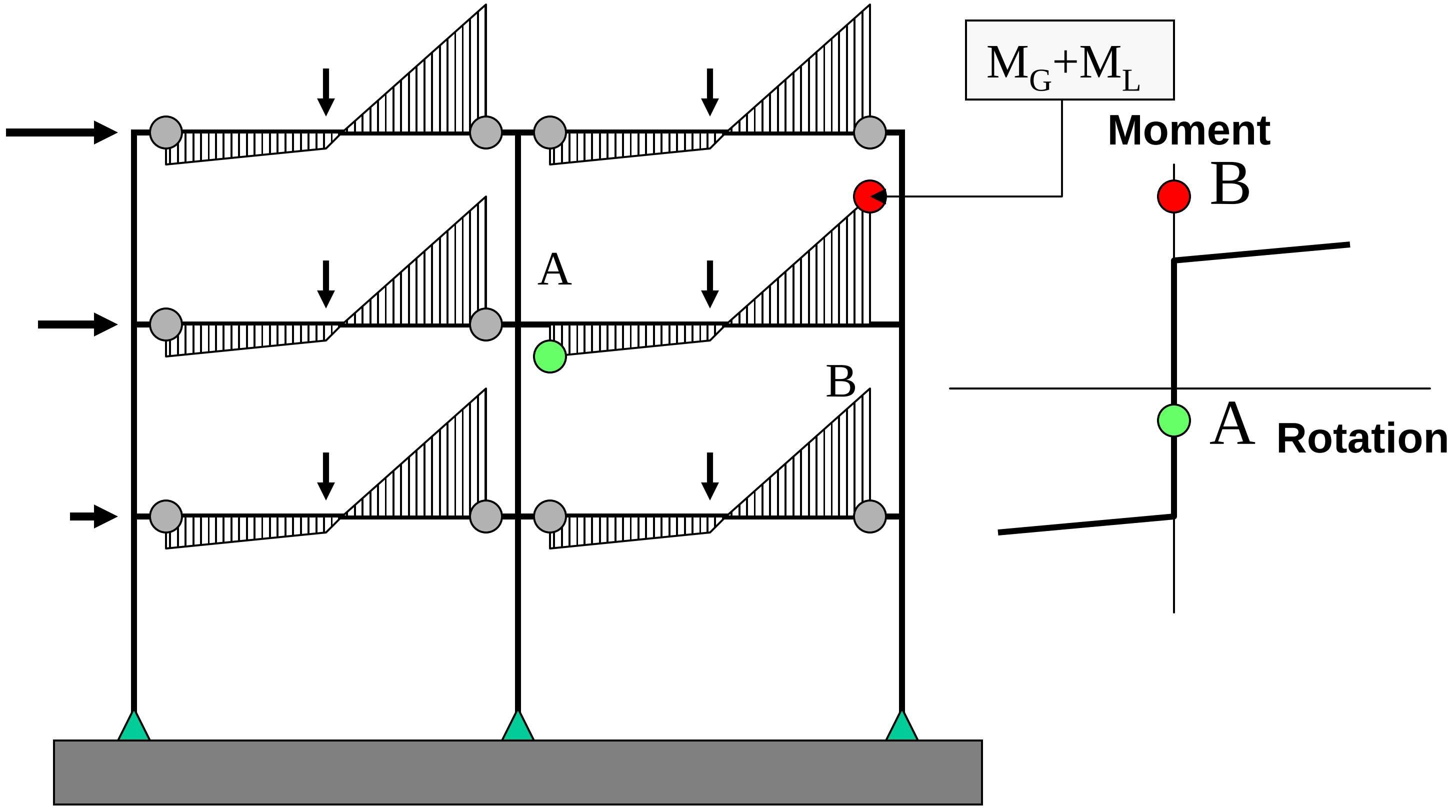
Lateral Load Analysis (Acting Alone)



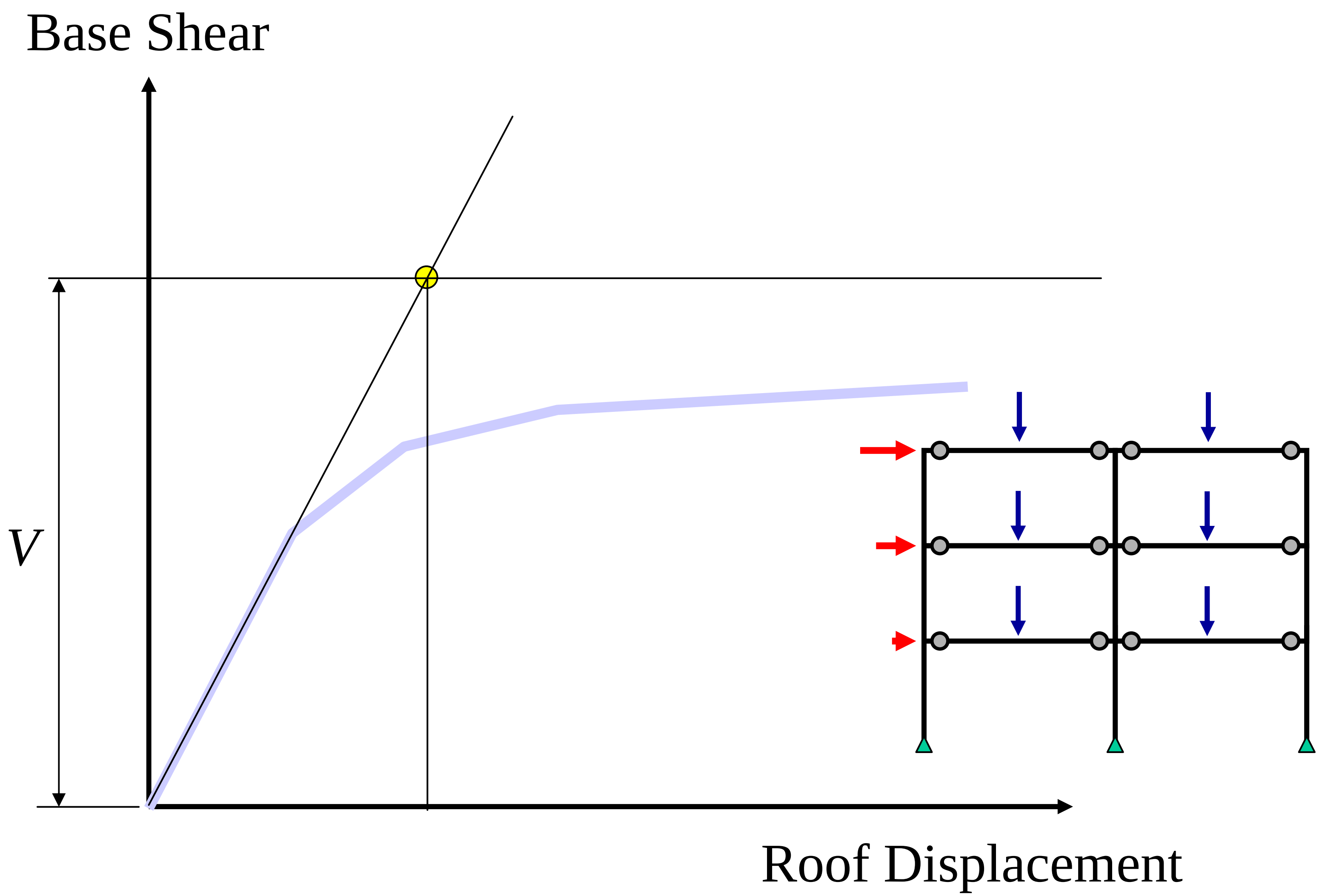
Moments plotted on tension side.

Combined Load Analysis Including Total Load V

Total
Load = V

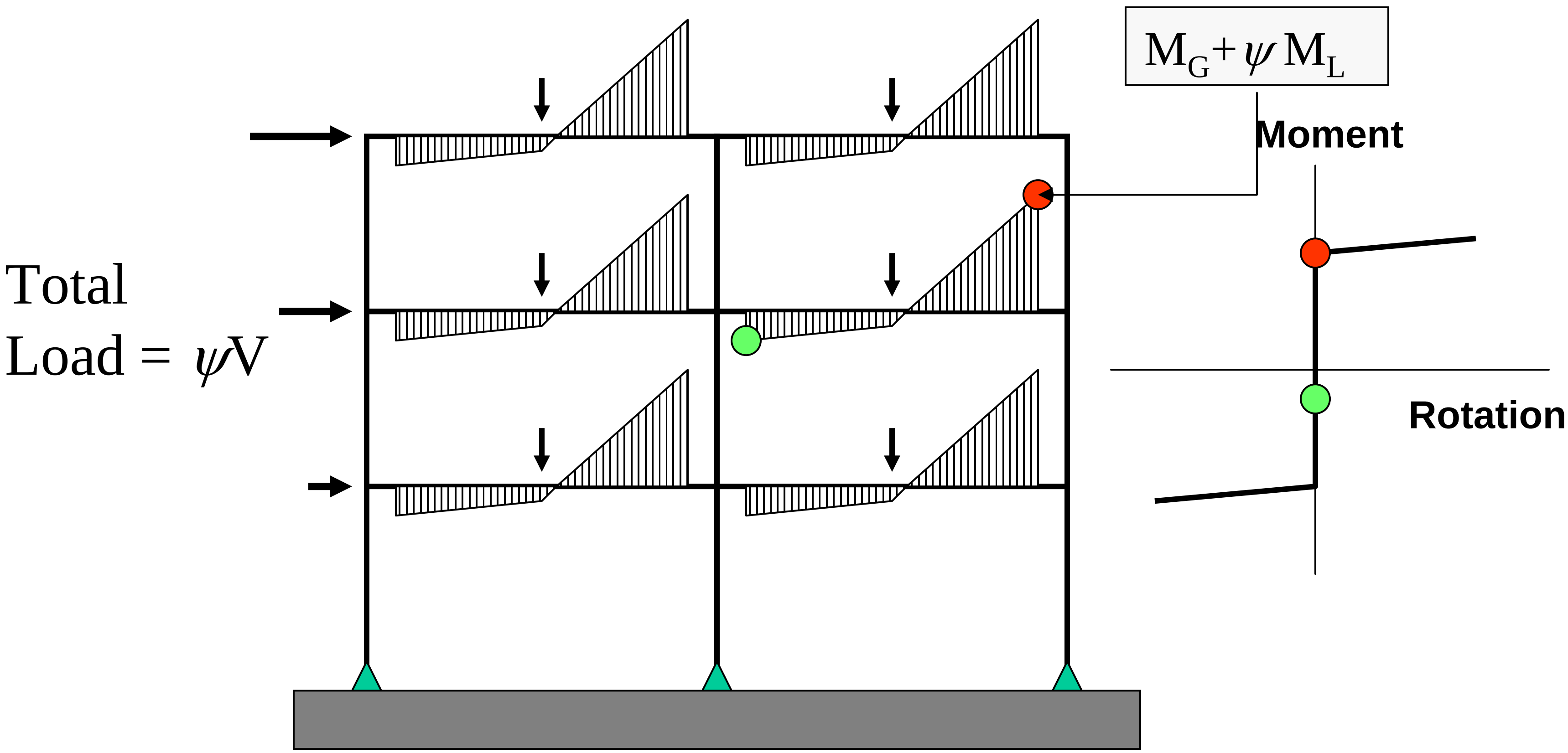


Analysis 2a First Lateral Analysis



Combined Load Analysis:

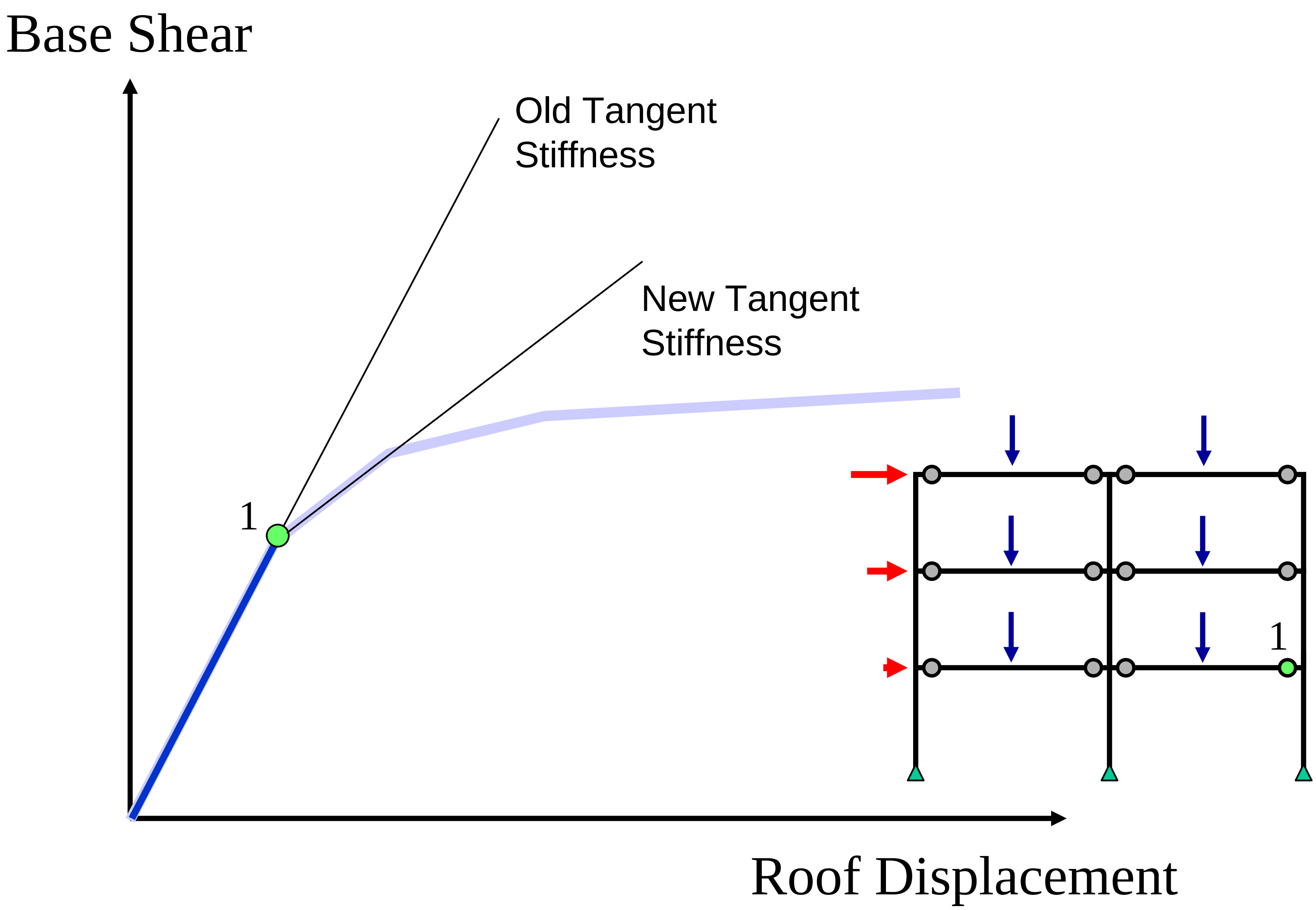
Determine amount of Lateral Load Required to Produce First Yield



For all potential hinges (i) find ψ_i such that

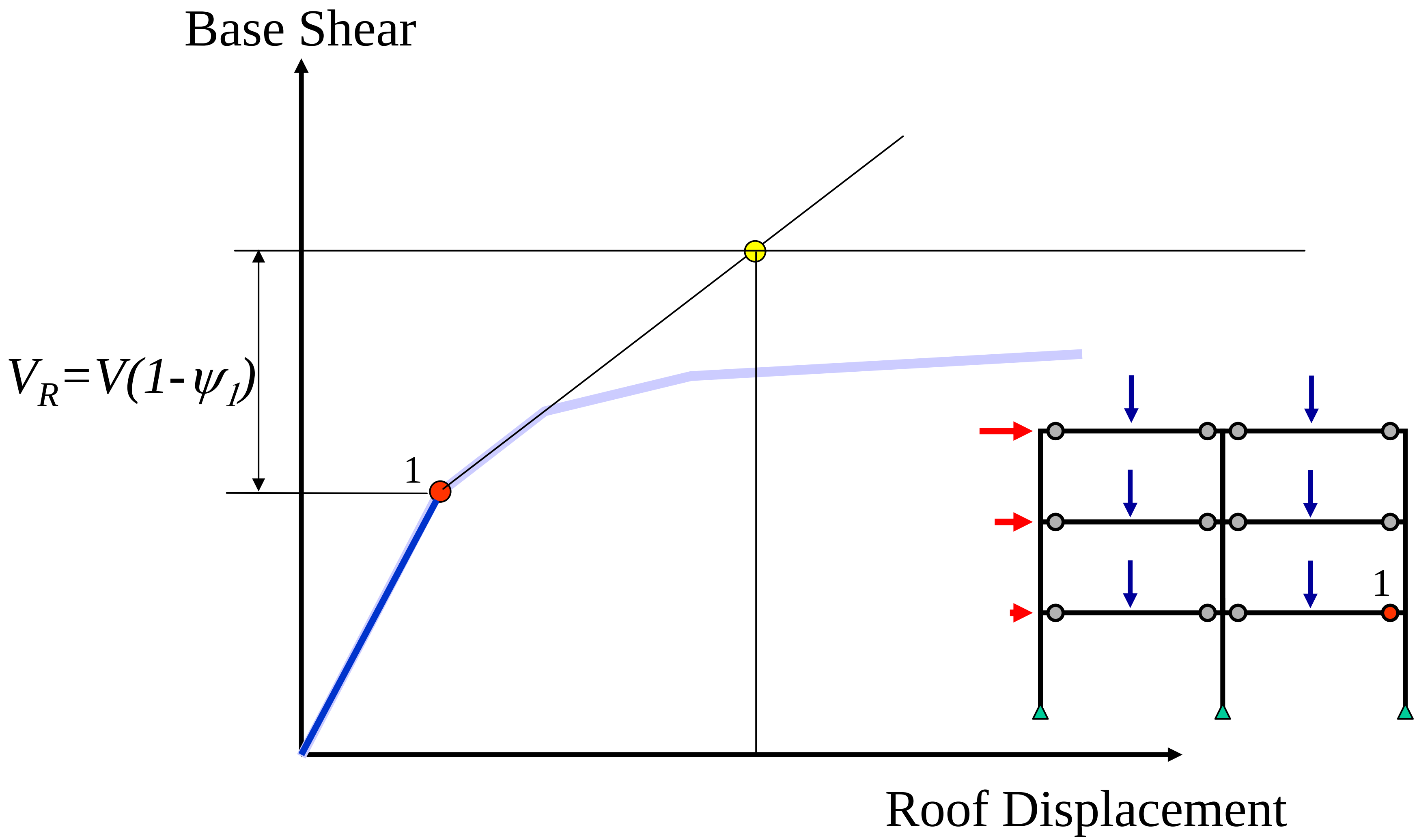
$$M_{G,i} + \psi_i M_{L,i} = M_{P,i}$$

Analysis 2b
Adjust Load to First Yield



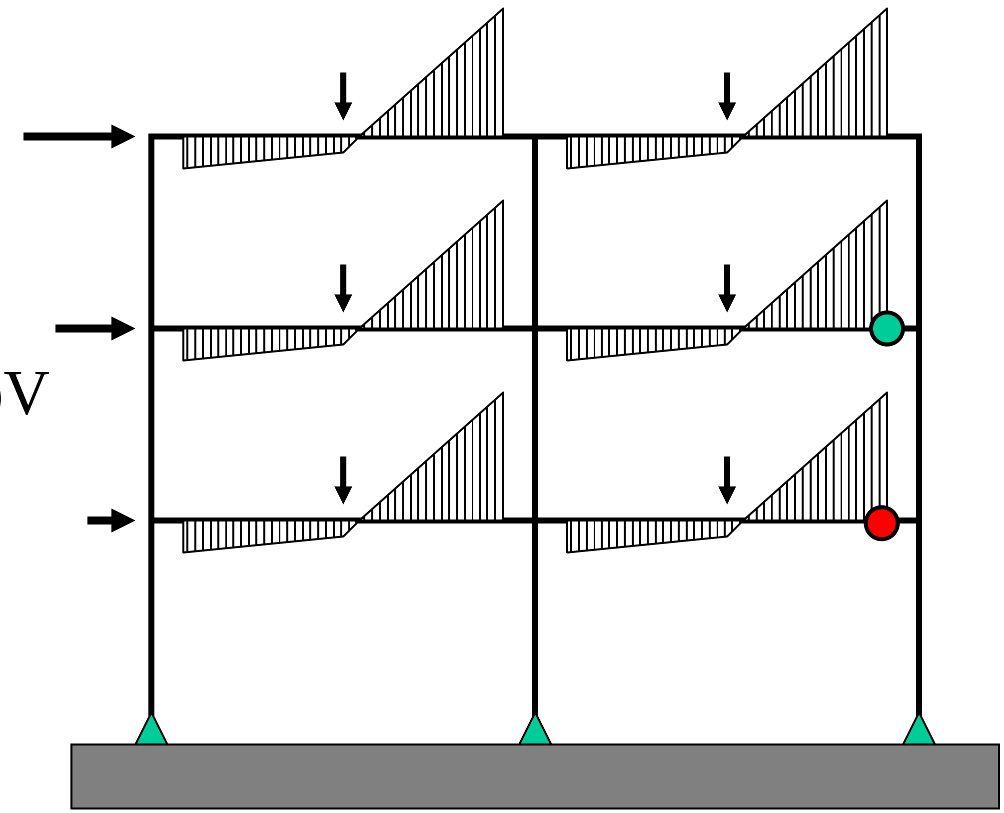
Analysis 3a

Modify System Stiffness Apply Remainder of Load

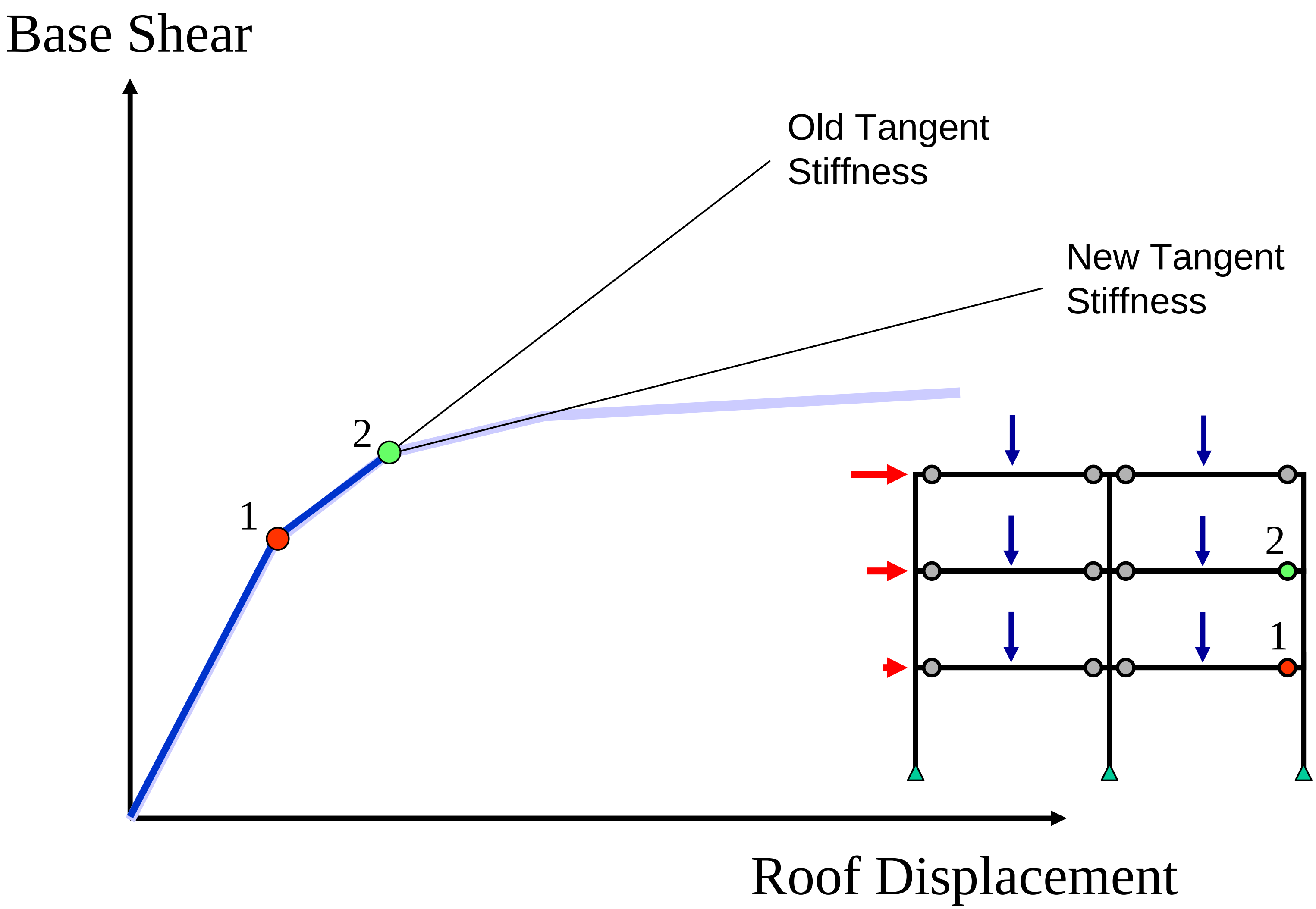


Determine amount of Lateral Load Required to Produce Second Yield

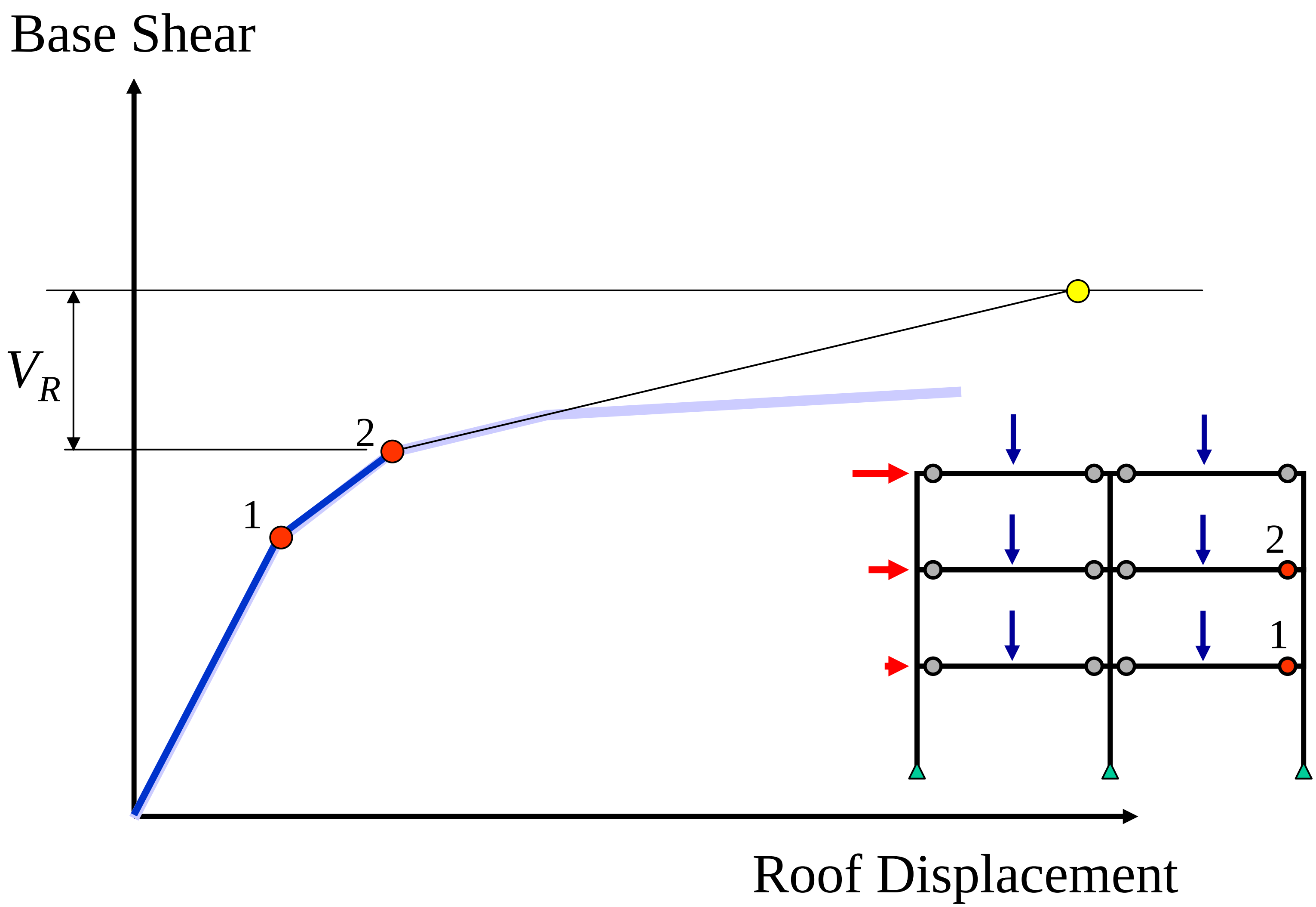
Total
Load = $(1-\psi_1)V$



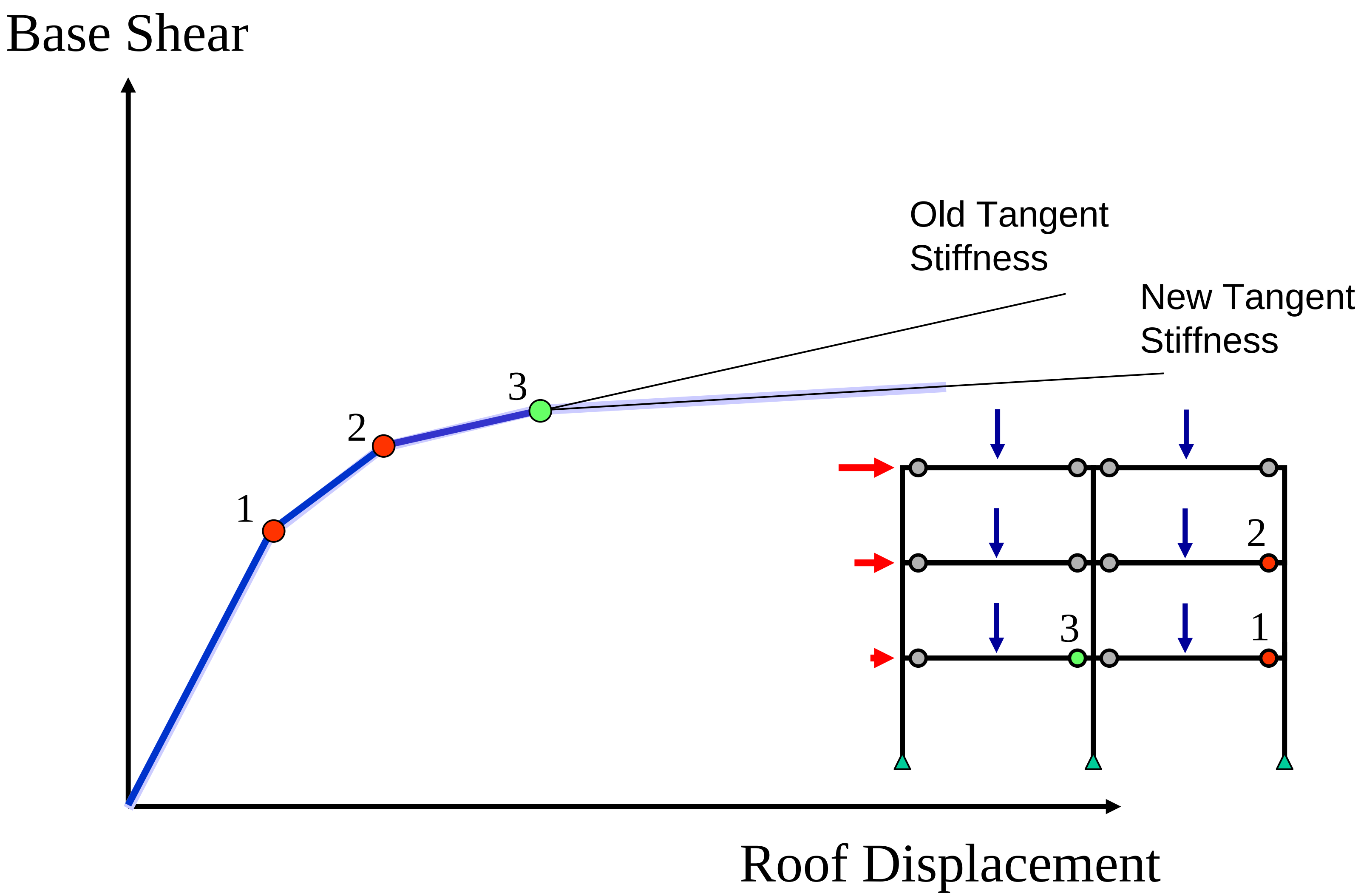
Analysis 3b
Adjust Load to Second Yield



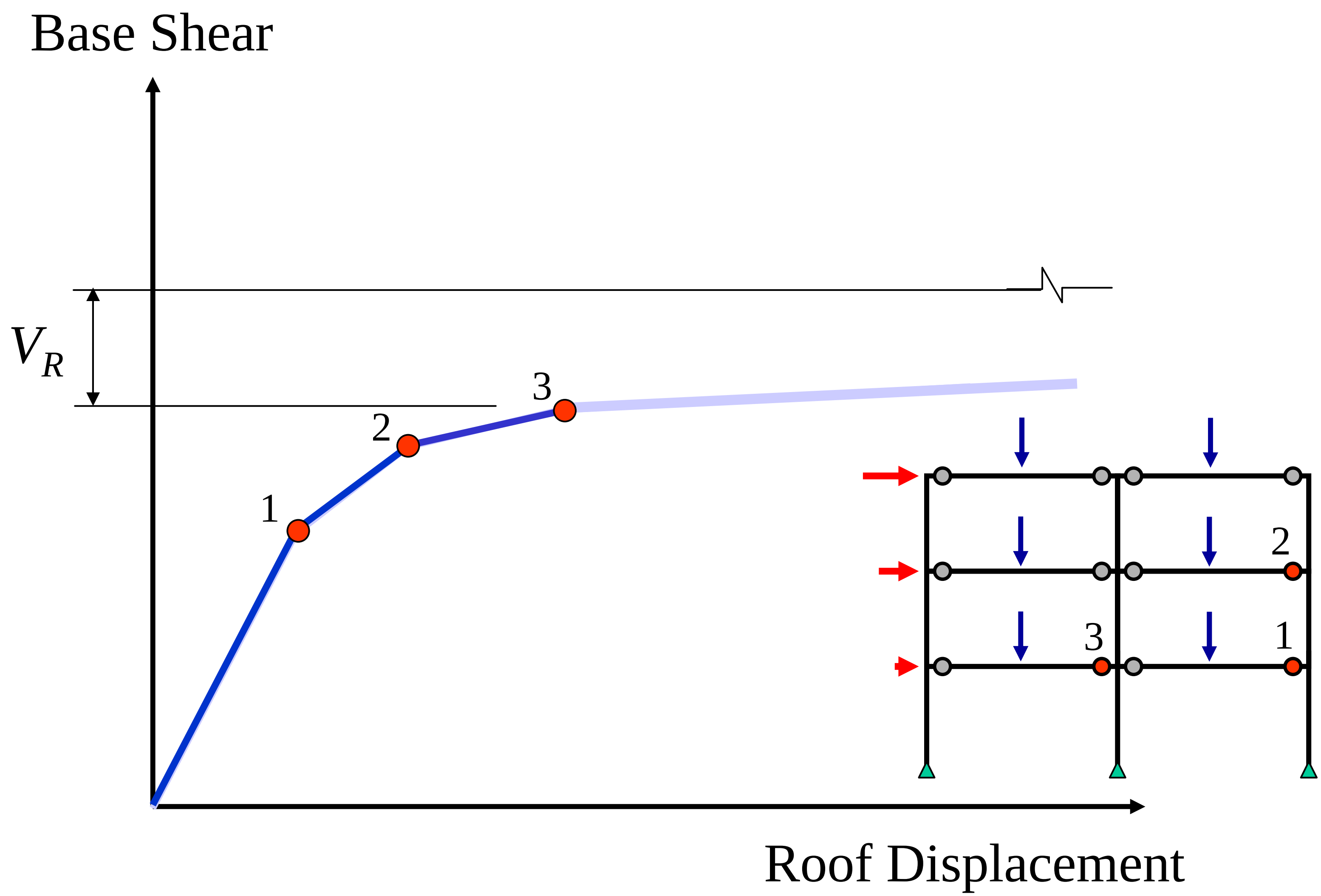
Analysis 4a
Modify System Stiffness Apply Remainder of Load



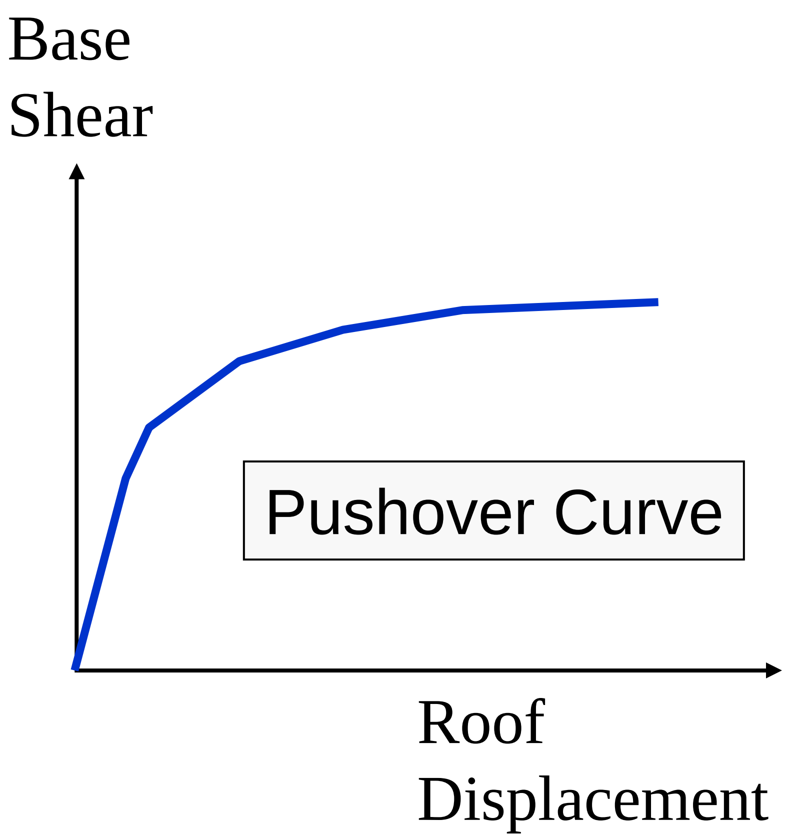
Analysis 4b
Adjust Load to Third Yield

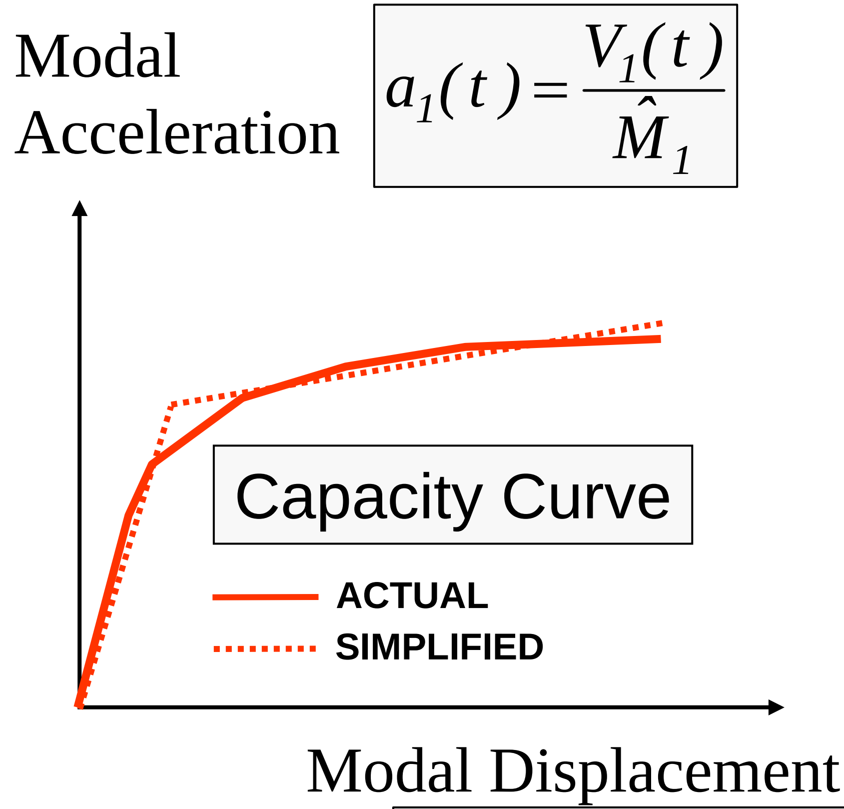


Analysis 5a…..



Convert Pushover Curve to Capacity Curve

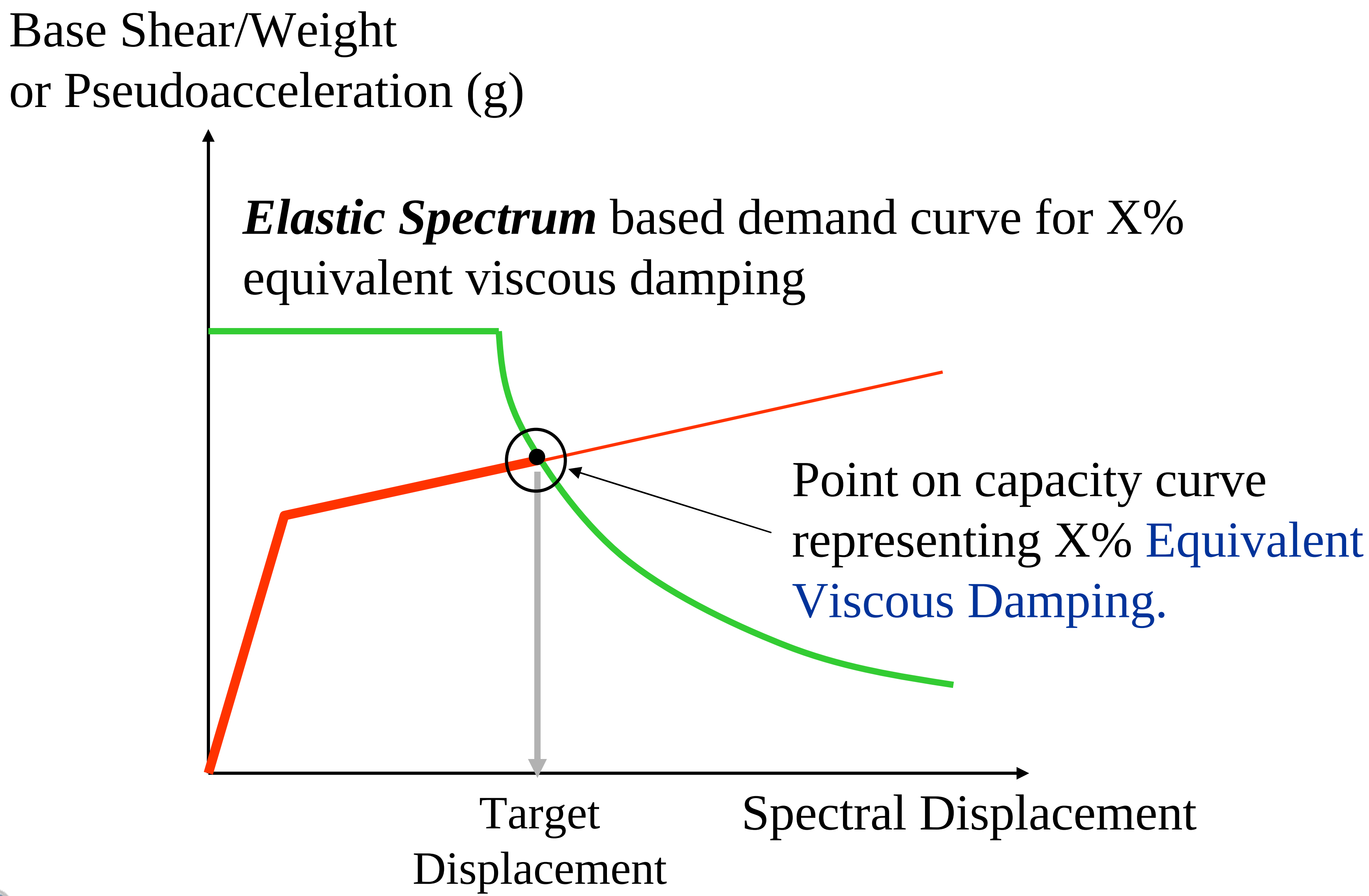




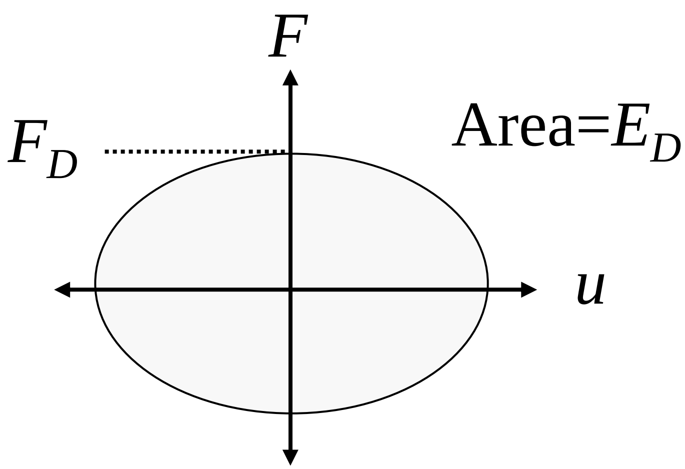
$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$

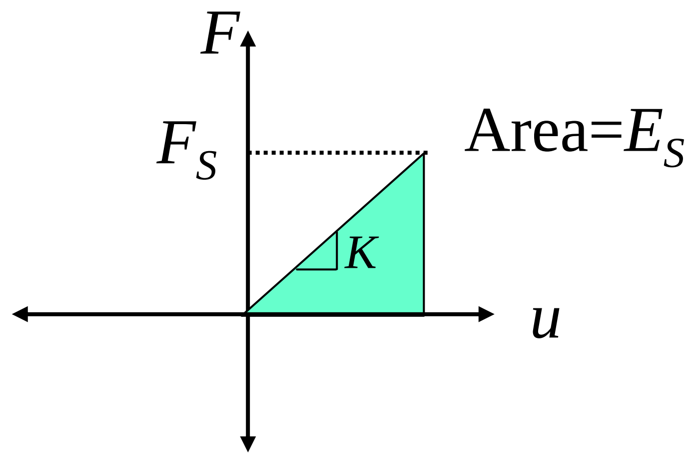
$$D_1(t) = \frac{u_1(t)}{\Gamma_1 \phi_{1,roof}}$$

Equivalent Viscous Damping



Computing Damping Ratio from Damping Energy and Strain Energy



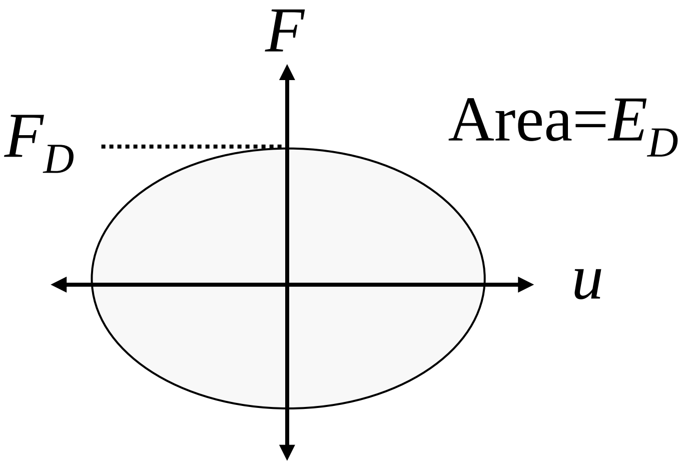


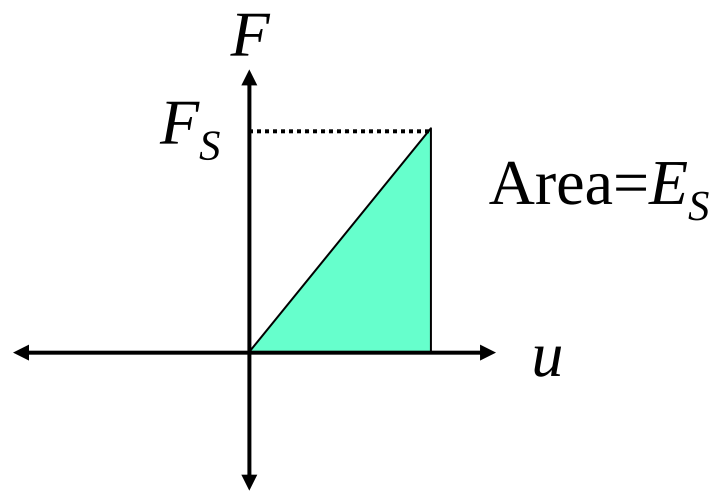
$$E_D = \pi F_D u = \pi C u^2 \omega = 2\pi \xi m \omega^2 u^2$$

$$E_S = 0.5 F_s u = 0.5 K u^2 = 0.5 m \omega^2 u^2$$

$$\xi = \frac{E_D}{4\pi E_S}$$

Computing Damping Ratio from Damping Force and Elastic Force





$$E_D = \pi F_D u$$

$$E_S = \frac{1}{2} F_s u$$

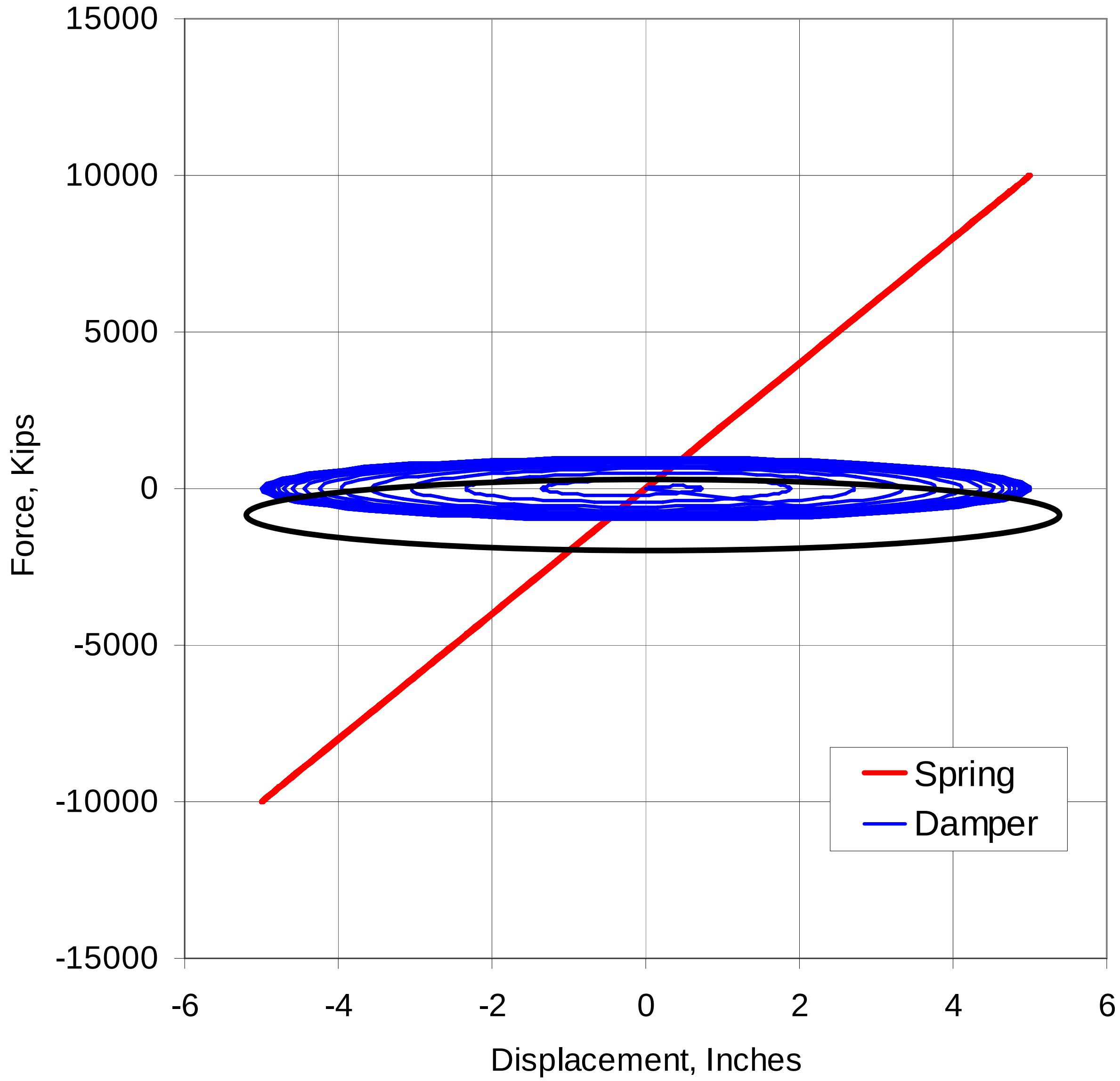
$$\xi = \frac{E_D}{4\pi E_S} = \frac{F_D}{2F_S}$$

Computing “True” Viscous Damping Ratio from Damping Energy and Strain Energy

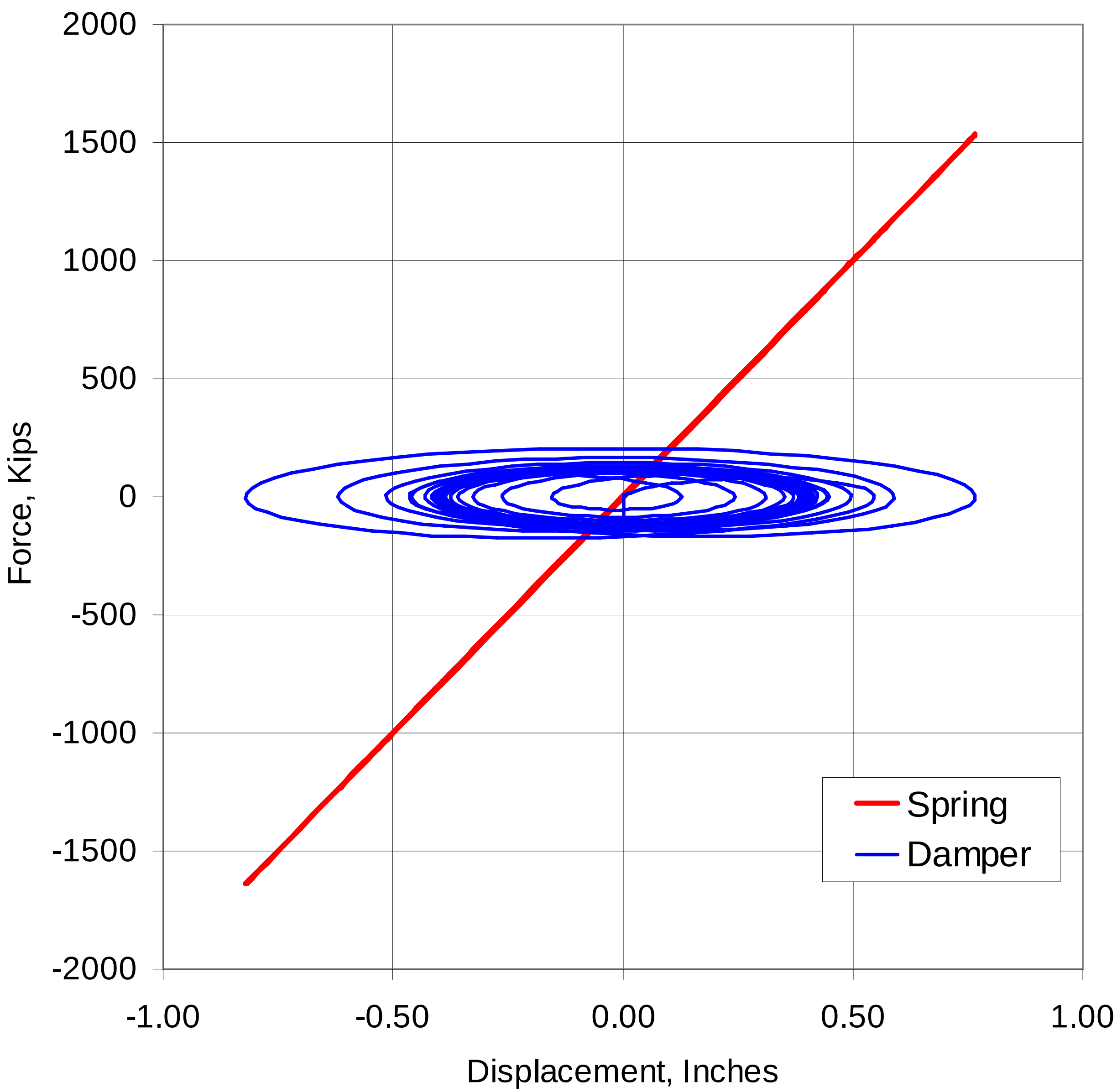
$$\xi = \frac{E_D}{4\pi E_S} = \frac{F_D}{2F_S}$$

Note:
System must be in steady state harmonic **RESONANT** response for this equation to work.

Harmonic Resonant Response from NONLIN



Harmonic Non-Resonant Response from NONLIN



Results from NONLIN Using: $\xi = \dfrac{E_D}{4\pi E_S} = \dfrac{F_D}{2F_S}$

System Period = 0.75 seconds
Harmonic Loading
Target Damping Ratio 5% Critical

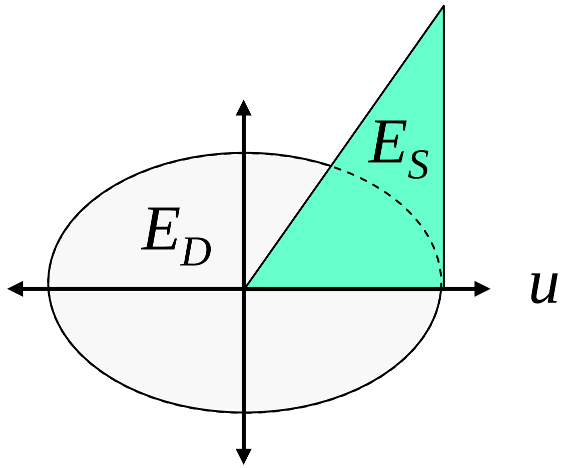
Loading Period (sec)	Damping Force (k)	Spring Force (k)	Damping Ratio %		
0.50	118	787	**7.50**	X	
0.75	984	9828	**5.00**	✓	**Resonant**
1.00	197	2251	**3.75**	X	

Results from NONLIN Using: $\xi = \dfrac{E_D}{4\pi E_S} = \dfrac{F_D}{2F_S}$

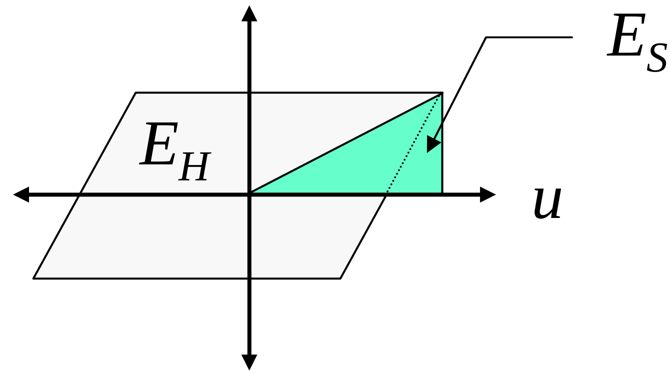
System Period = 0.75 seconds
Harmonic Loading
Target Damping Ratio 20% Critical

Loading Period (sec)	Damping Force (k)	Spring Force (k)	Damping Ratio %	
0.50	430	717	**30.0** ✗	
0.75	999	2498	**20.0** ✓	**Resonant**
1.00	1888	5666	**16.7** ✗	

Computing *Equivalent* Viscous Damping Ratio from Yield-Based Hysteretic Energy and Strain Energy



Viscous System

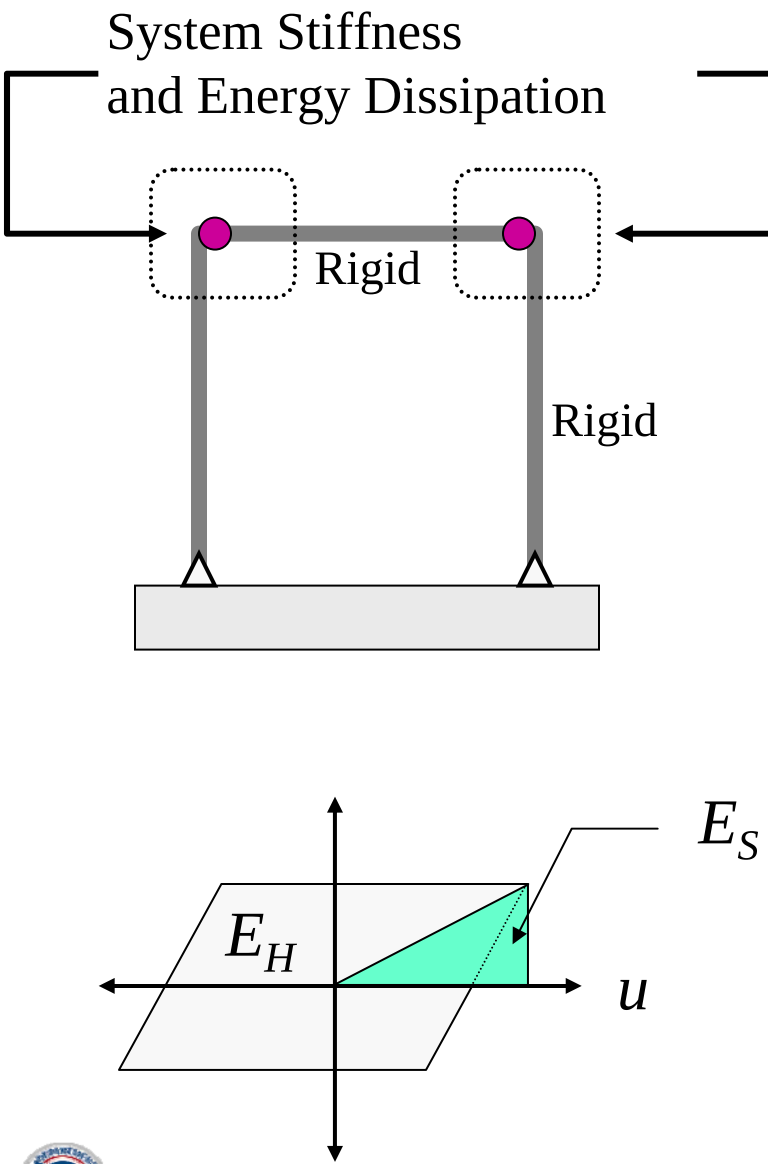


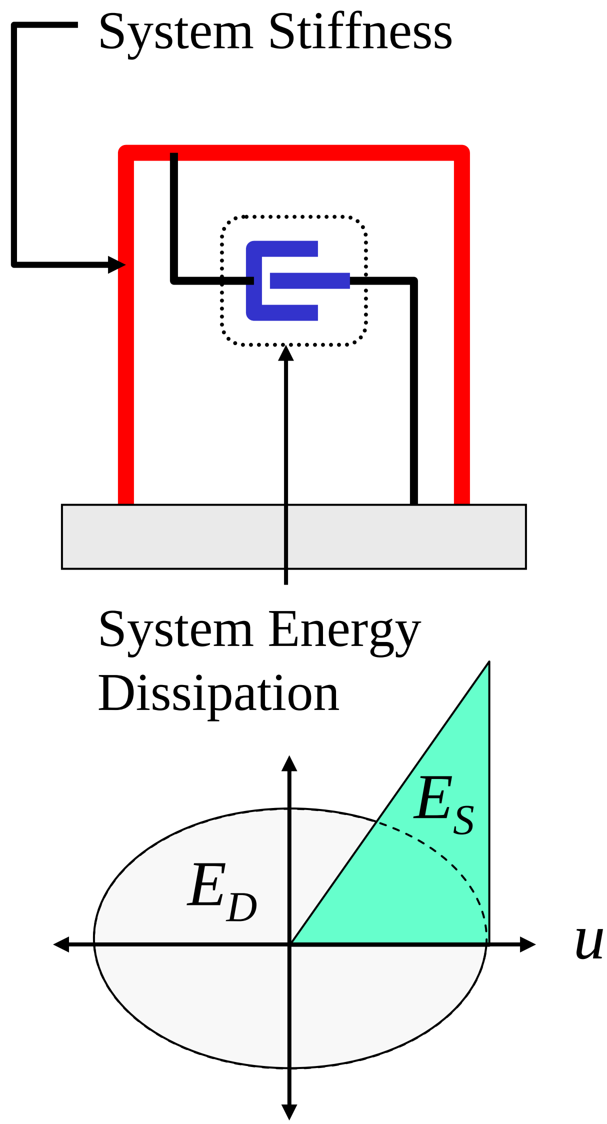
Yielding System

$$\xi \equiv \frac{E_H}{4\pi E_S}$$

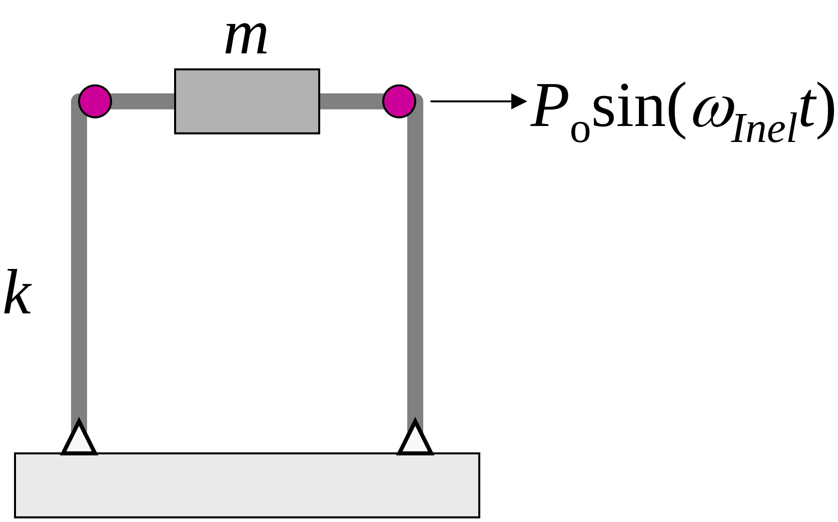
Actual Yielding System

“Equivalent” Elastic System

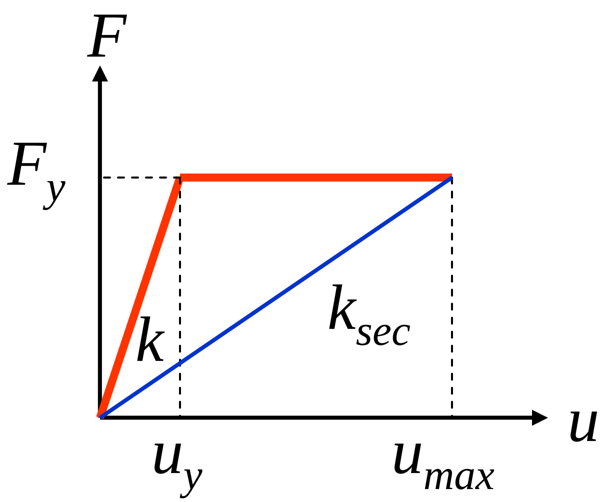




Original Yielding System



Initial Stiffness k



Initial Frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

Resonant Frequency:

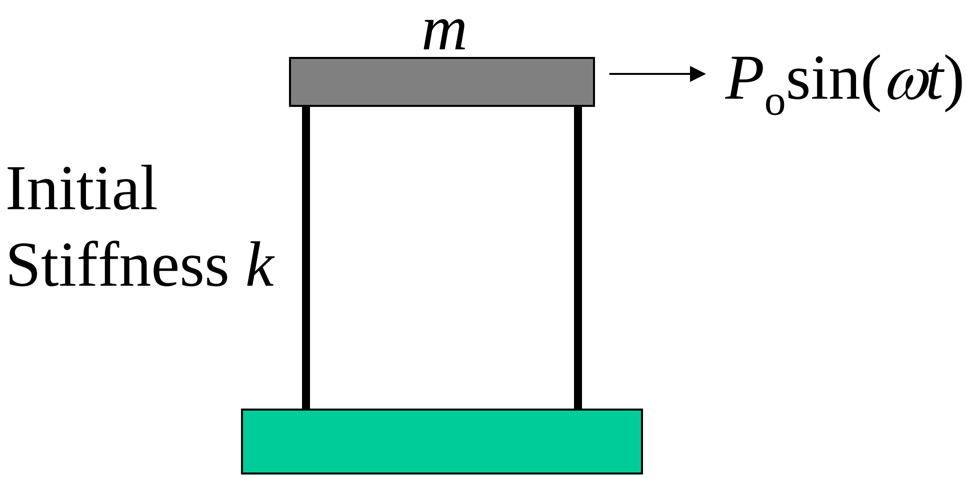
$$\omega_{Inelastic} = \frac{\omega^2}{\pi}(\theta - 0.5\sin 2\theta)$$

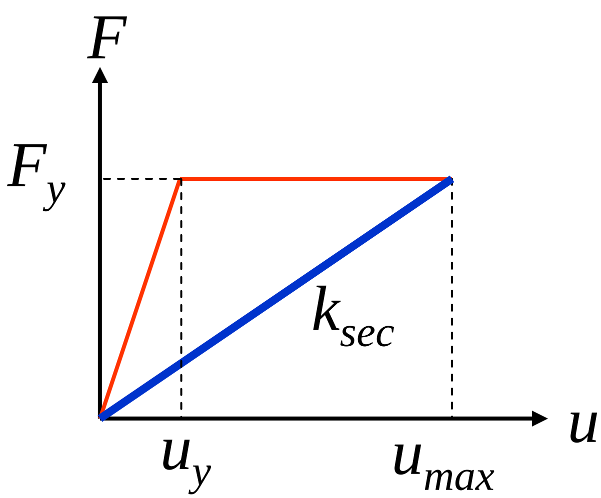
$$\theta = \cos^{-1}(1 - 2\frac{u_y}{u_{\max}})$$

Maximum Steady State Response (loaded at $\omega_{Inelastic}$):

$$u_{\max} = \frac{4u_y}{4 - \dfrac{P_o \pi}{k u_y}}$$

"Equivalent" Elastic System





Resonant Frequency:

$$\omega_{\sec} = \sqrt{\frac{k_{\sec}}{m}}$$

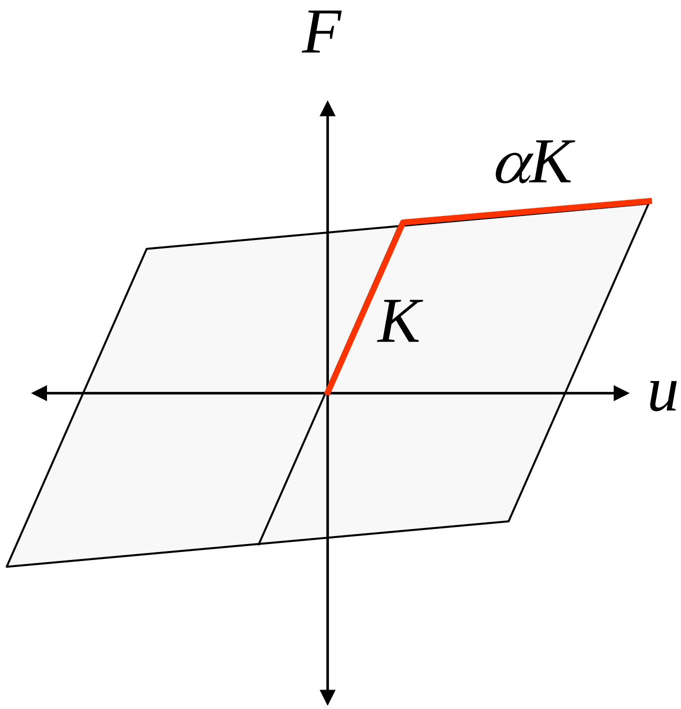
Maximum Steady State Resonant Response:

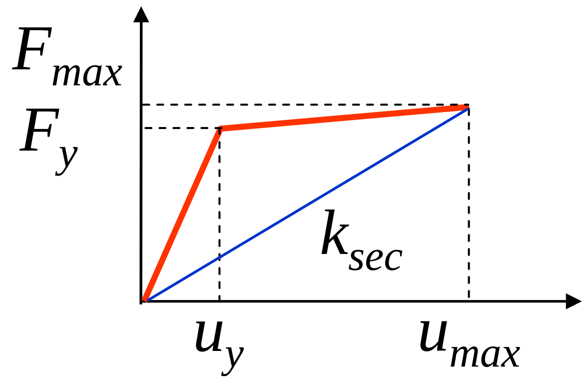
$$u_{\max} = \frac{P_o}{2\xi_{\sec} k_{\sec}}$$

Equivalent Damping:

$$\xi_{\sec} = 0.637\left(1 - \frac{u_y}{u_{\max}}\right) = 0.637\left(1 - \frac{1}{\mu_{\Delta}}\right)$$

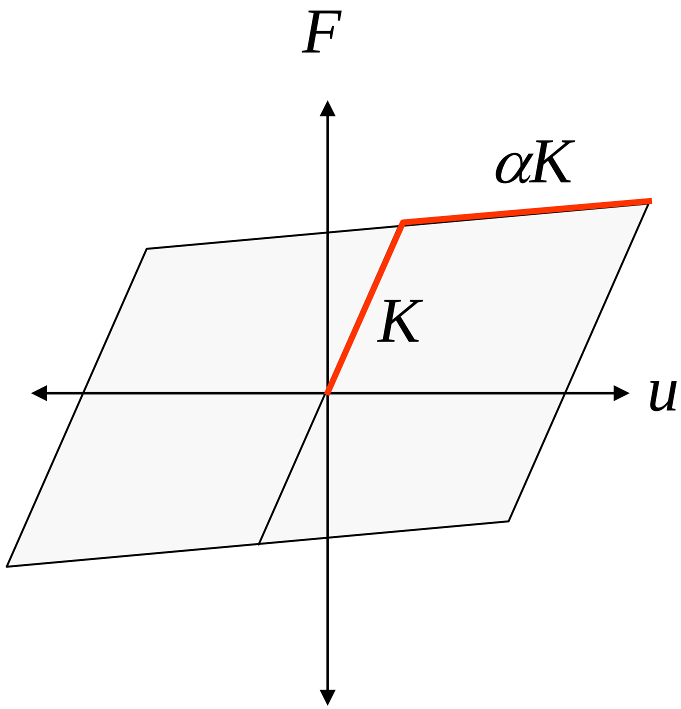
“Equivalent” Elastic System when Strain Hardening is Included

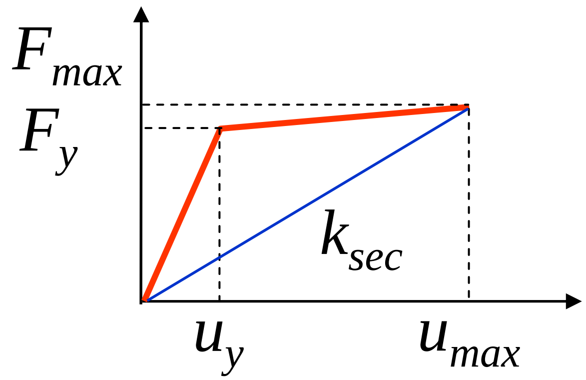




$$\xi_{\sec} \equiv 0.637 \frac{(F_y u_{\max} - F_{\max} u_y)}{F_{\max} u_{\max}}$$

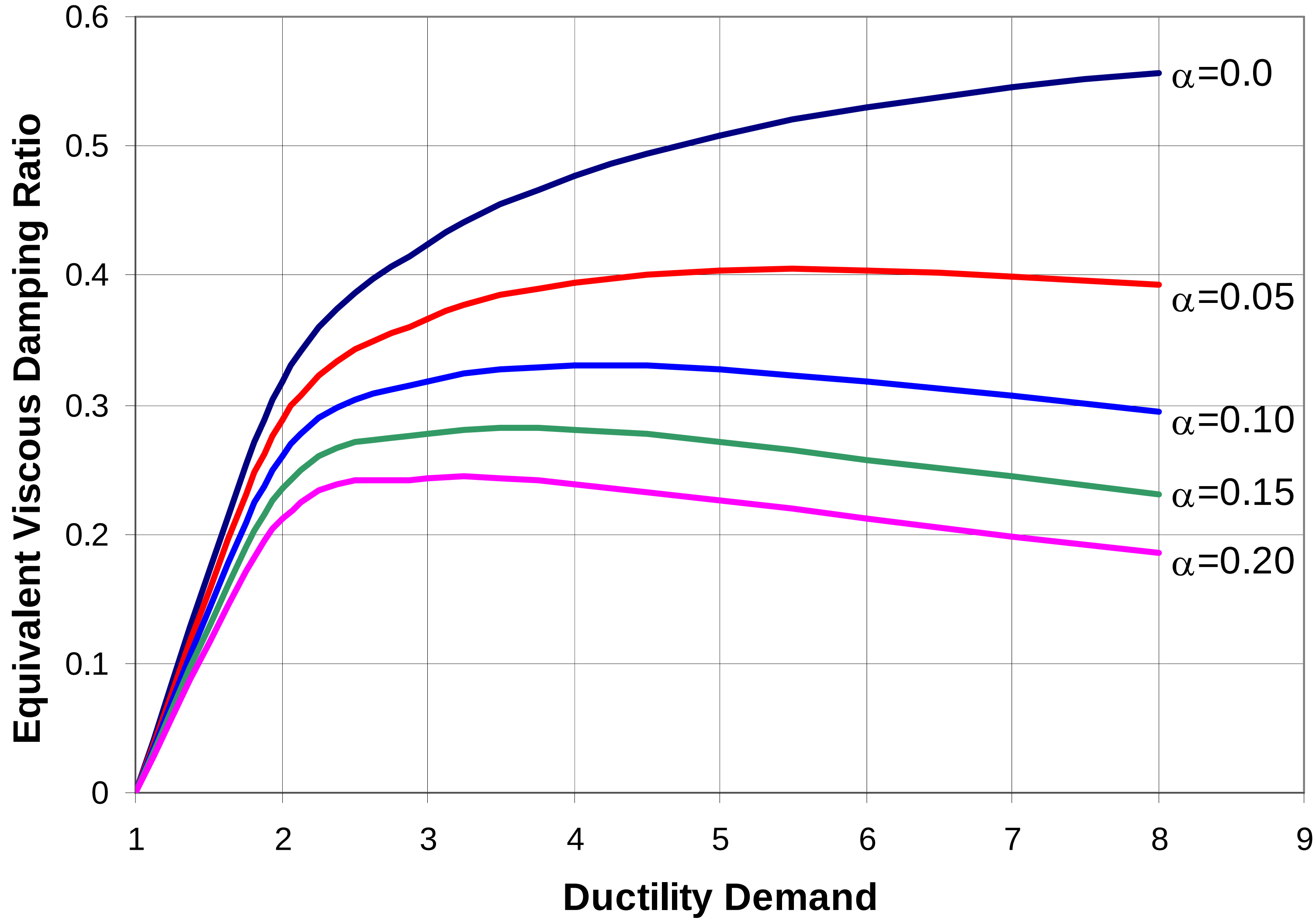
“Equivalent” Elastic System when Strain Hardening is Included





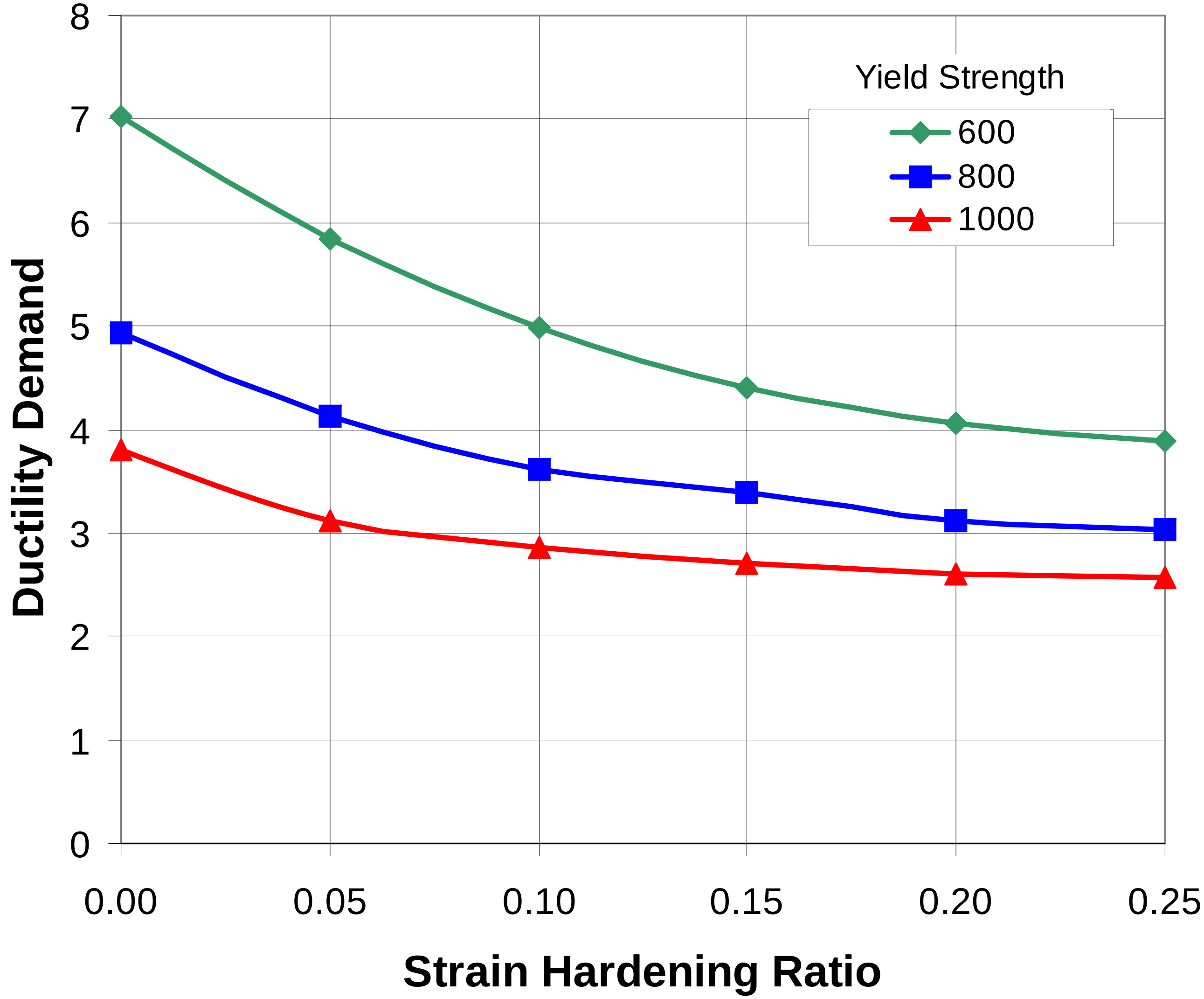
$$\xi_{Equiv} \equiv 0.637\left[\frac{F_y}{F_{\max}} - \frac{u_y}{u_{\max}}\right] = 0.637\left[\frac{1}{\alpha(\mu_\Delta - 1) + 1} - \frac{1}{\mu_\Delta}\right]$$

Effect of Secondary Stiffness On Equivalent Viscous Damping



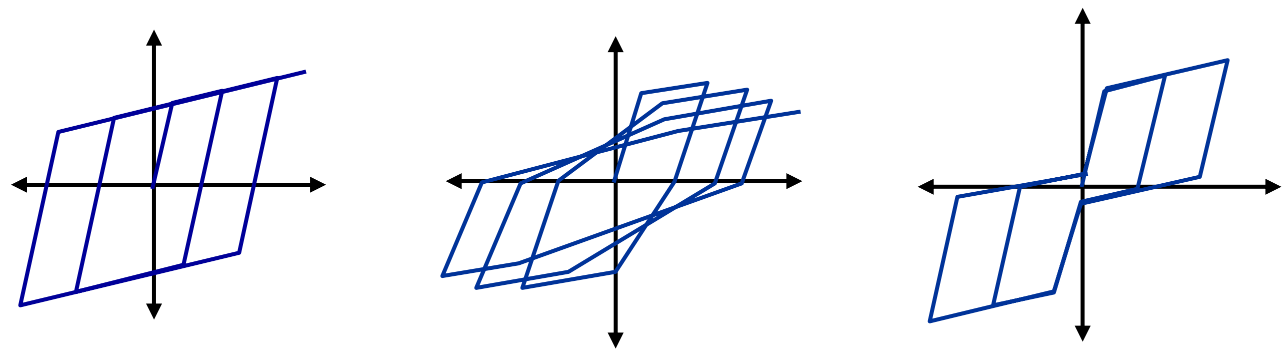
Reduction in Ductility Demand with Strain Hardening Ratio

(W = 11250 k, K = 918 k/in., T=1.0 sec, El Centro Ground Motion)



Total System Damping (% Critical)

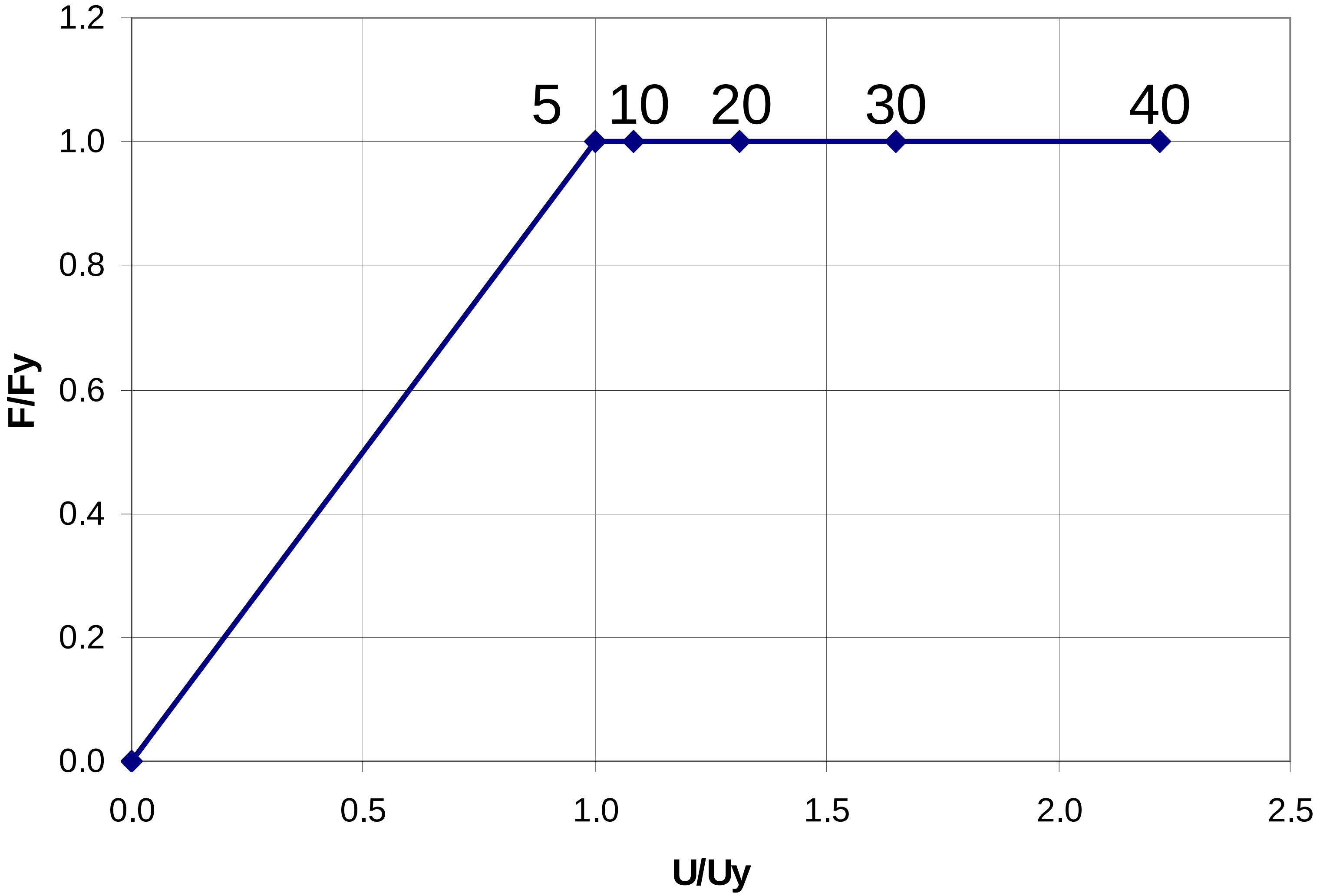
$$\xi_{Total} = 5 + \kappa \xi_{Equiv}$$



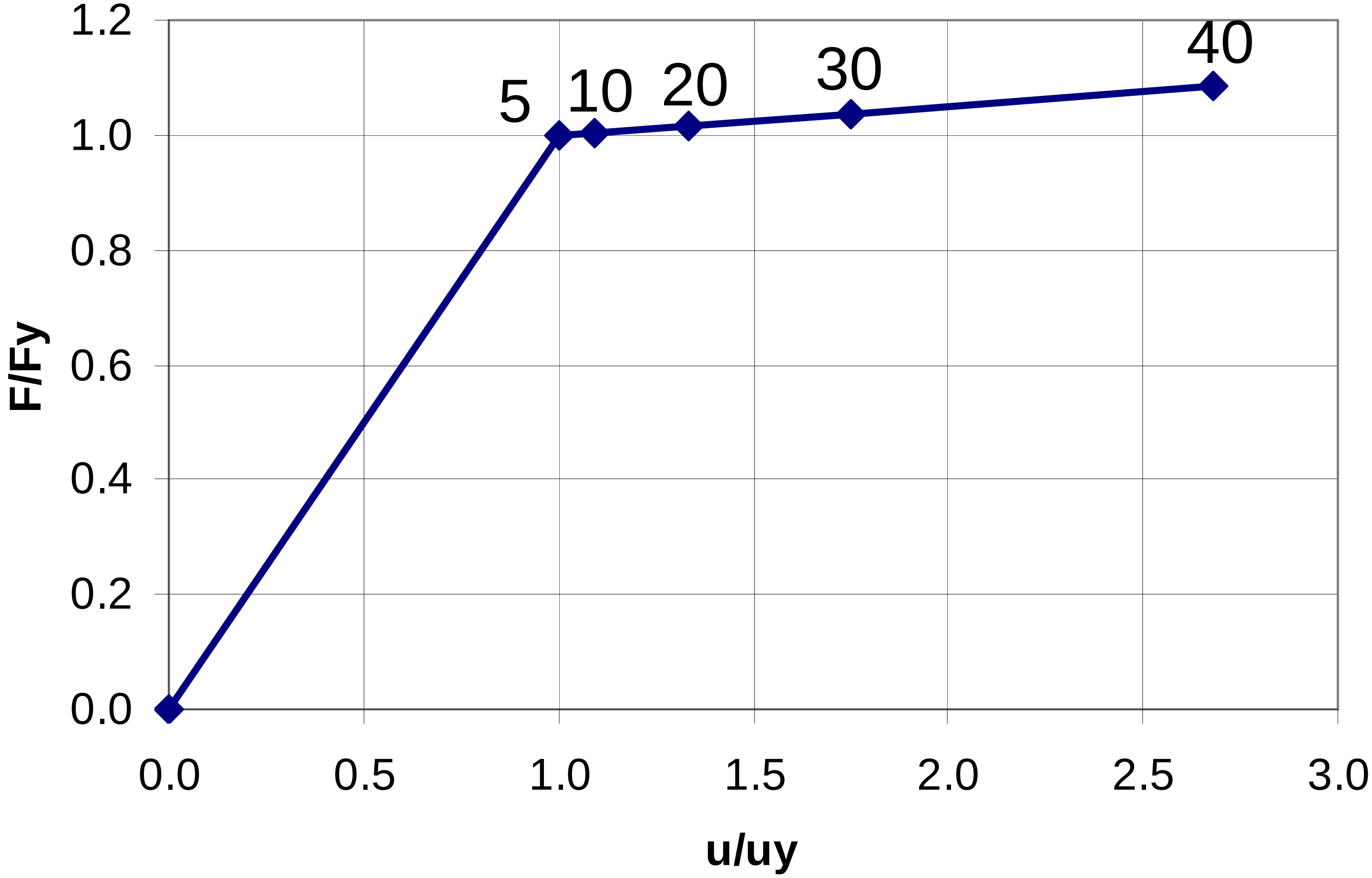
Shaking Duration	Robust	Moderately Robust	Pinched Or Brittle
Short	$\kappa = 1$	$\kappa = .7$	$\kappa = .7$
Long	$\kappa = .7$	$\kappa = .33$	$\kappa = .33$

See ATC 40 for Exact Values

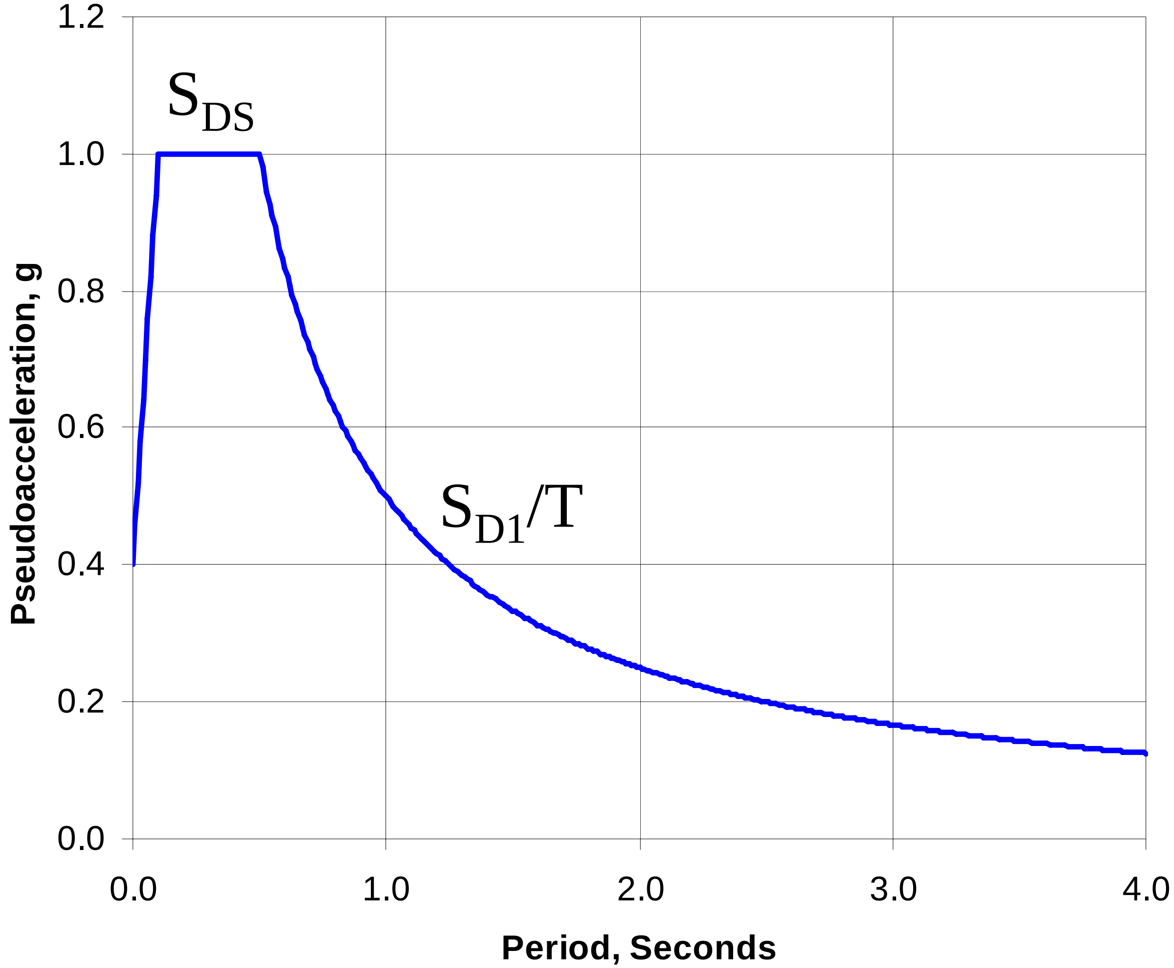
Equivalent Viscous Damping Values for EPP System
(Values Shown are Percent Critical)



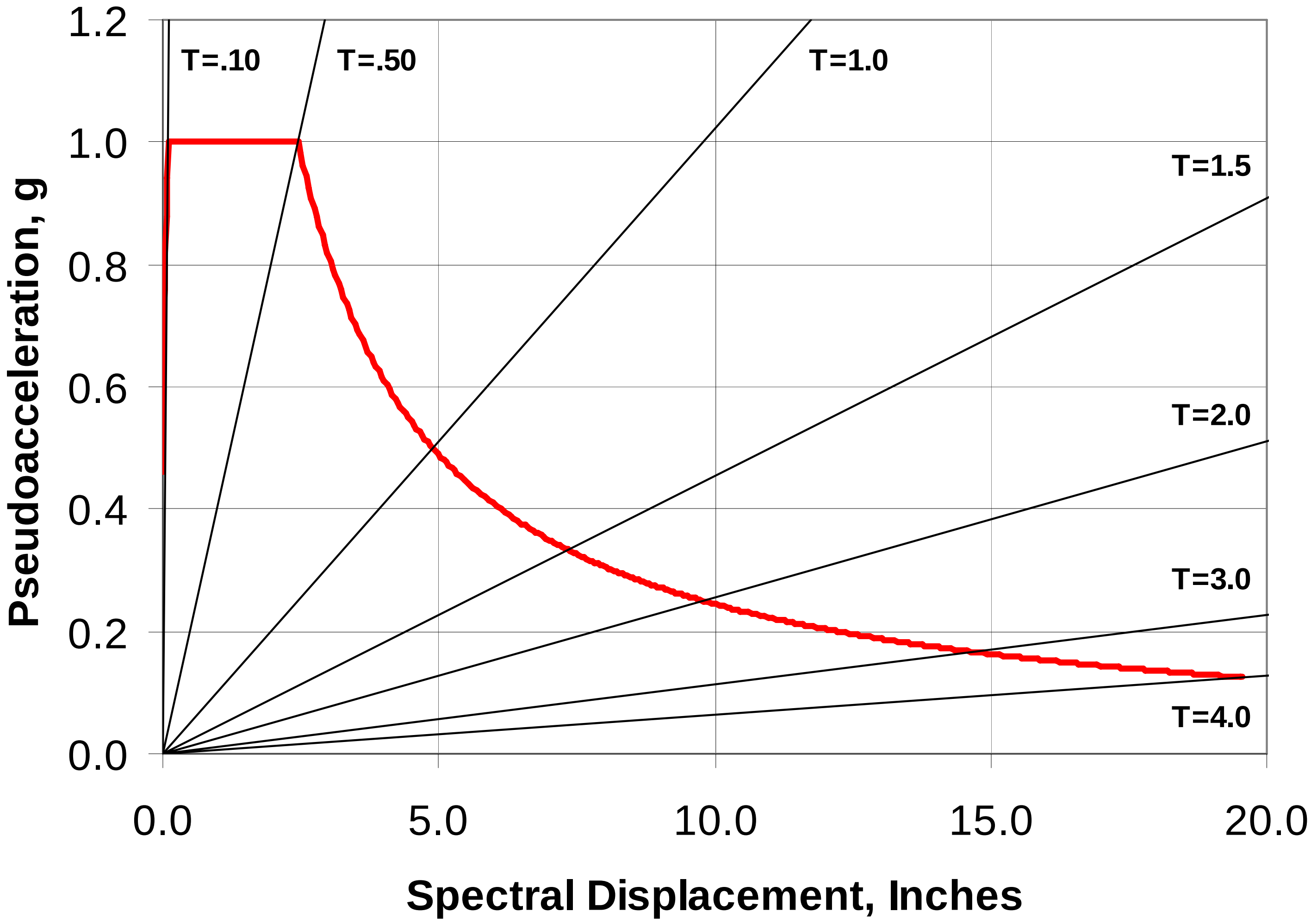
Equivalent Viscous Damping Values for System
With 5% Strain Hardening Ratio
(Values Shown are Percent Critical)



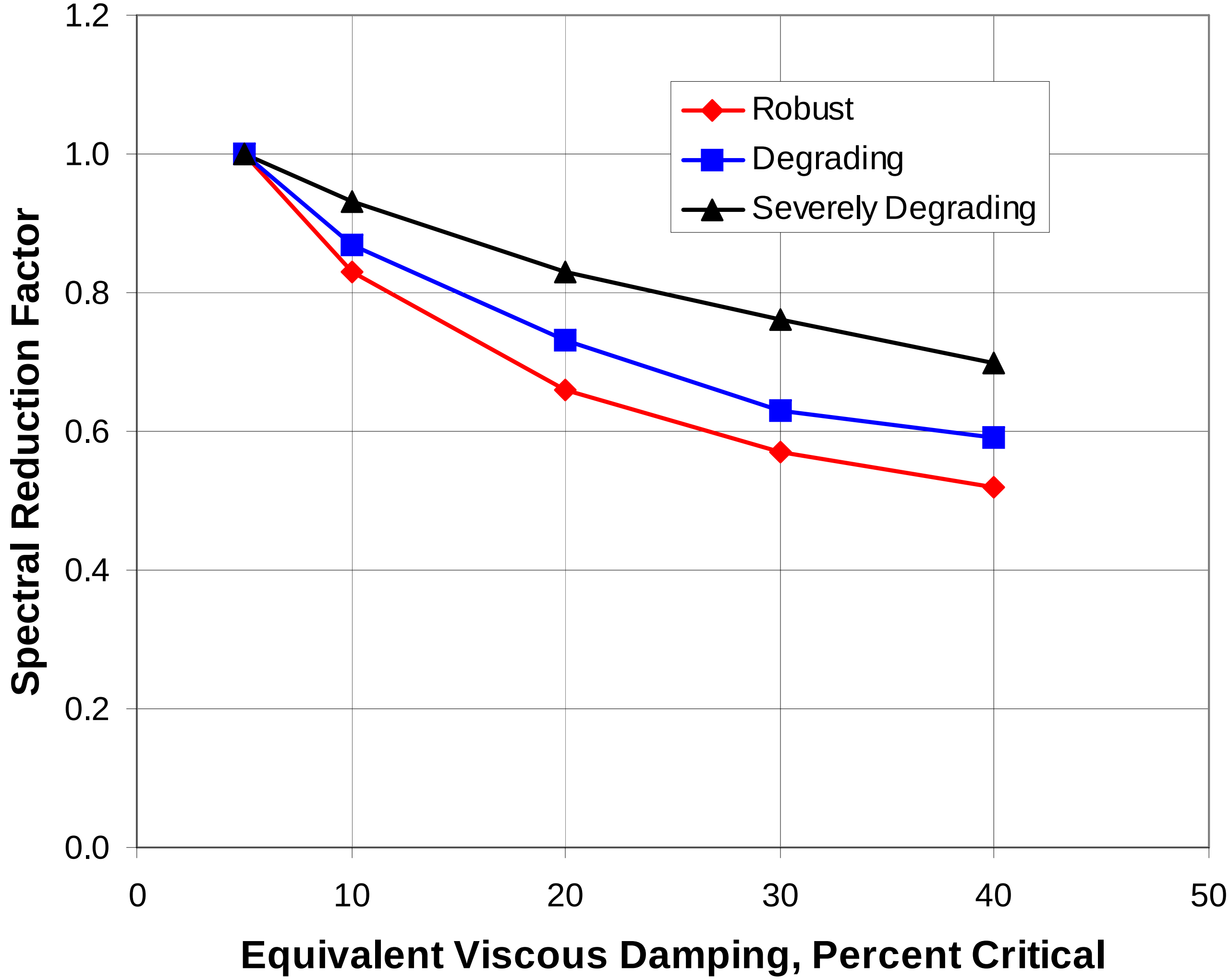
Pseudoacceleration Spectrum in Traditional Format



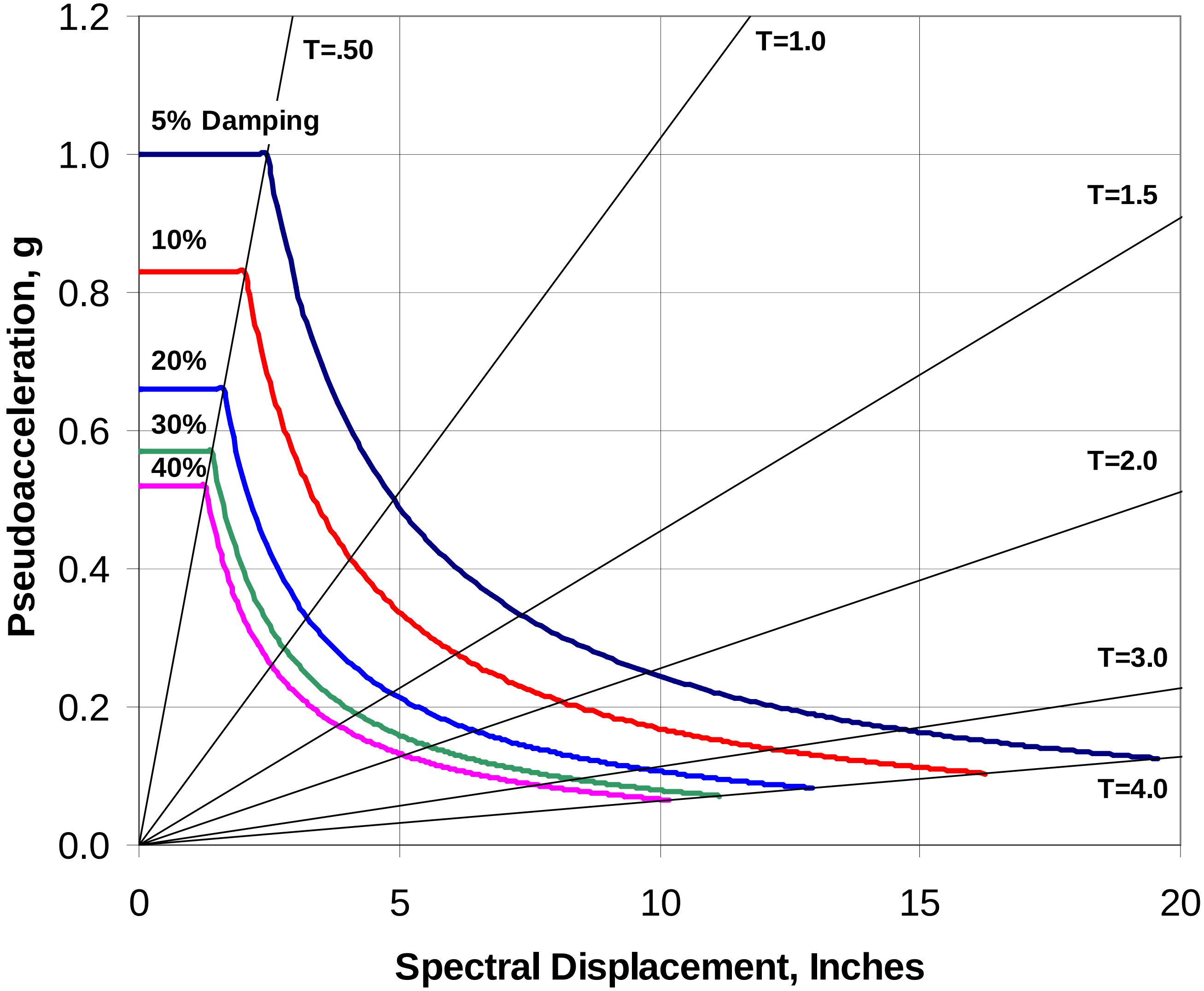
Pseudoacceleration (Demand) Spectrum in ADRS Format (5% Damping)



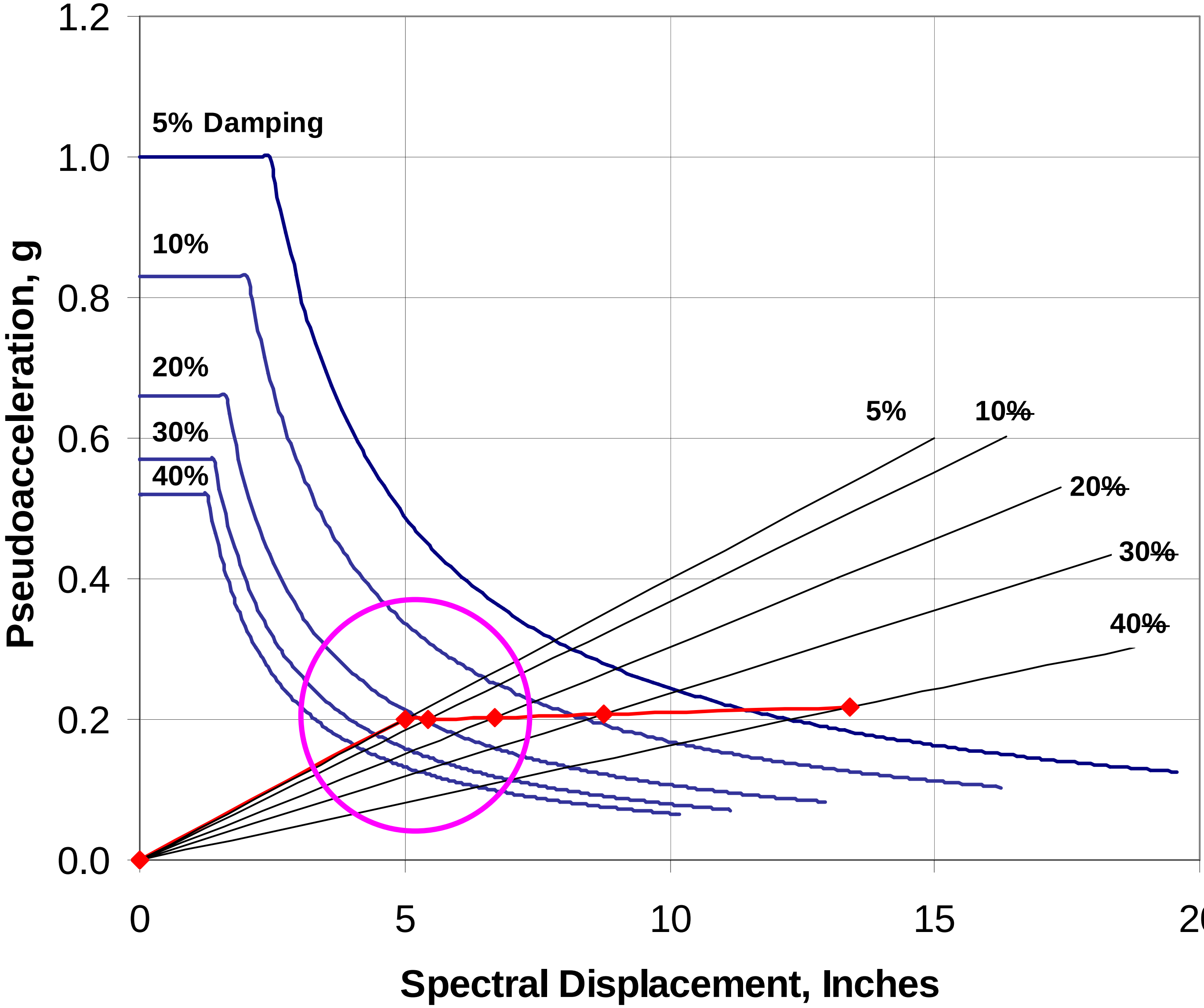
Spectral Reduction Factors for Increased Equivalent Damping



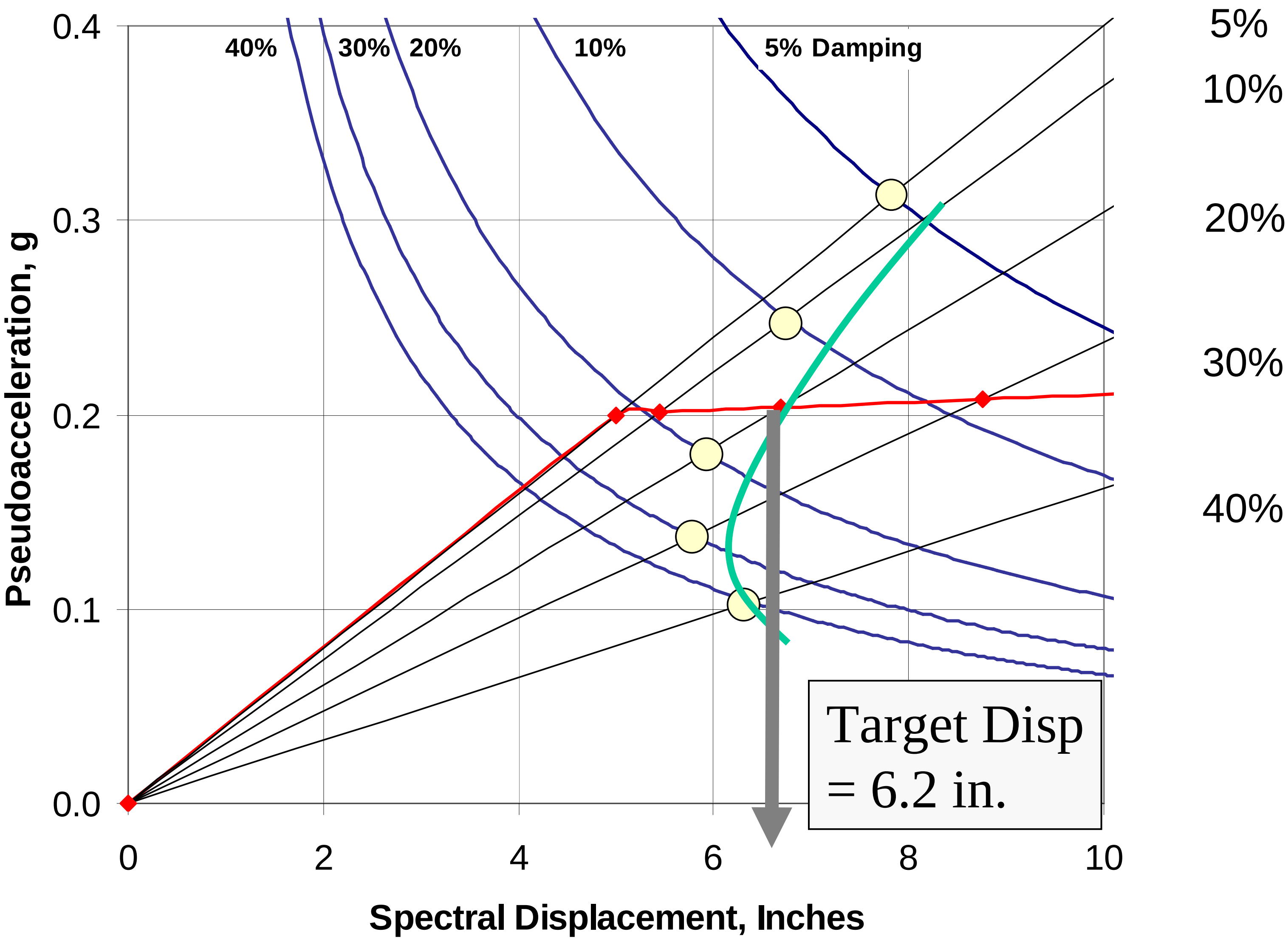
Demand Spectra for Various Damping Values



Combined Capacity-Demand Spectra



Finding the Target Displacement



You are Not Done Yet!

- Note: The target displacement from the Capacity-Demand diagram corresponds to a first mode SDOF system. It must be multiplied by the first mode modal participation factor and the modal amplitude of the first mode mode shape at the roof to determine displacements or deformations in the original system.

  Hinge rotations may then be obtained for comparison with performance criterion.

- Knowing the target displacement, the base shear can be found from the original pushover curve.

“There is sometimes cause to fear that scientific technique, that proud servant of engineering arts, is trying to swallow its master”

Professor Hardy Cross

Simplified Pushover Approaches: 2003 *NEHRP Recommended Provisions*

- Procedure is presented in Appendix to Chapter 5
- Gravity Loads include 25% of live load (but Provisions are not specific on P-Delta Modeling Requirements)
- Lateral Loads Applied in a "First Mode Pattern"
- Structure is pushed to 150% of target displacement
- Target displacement is assumed equal to the displacement computed from a first mode response spectrum analysis, multiplied by the factor C_i
- C_i adjusts for "error" in equal displacement theory when structural period is low

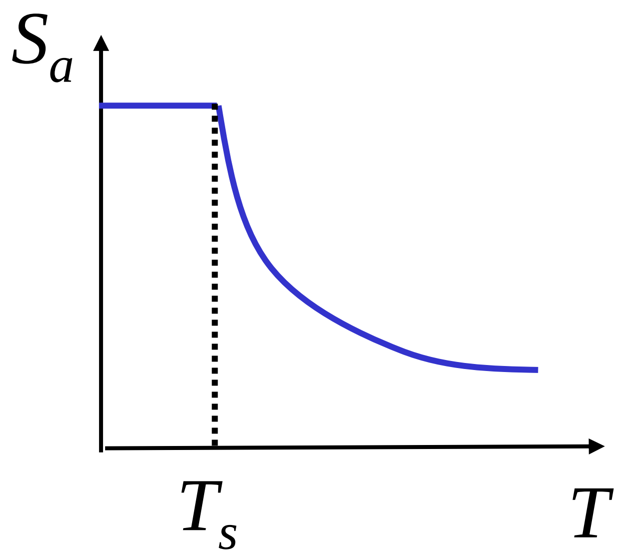
Simplified Pushover Approaches: 2003 *NEHRP Provisions* (2)

$$C_i = \frac{(1 - T_s / T_1)}{R_d} + (T_s / T_1)$$

C_i=1 if T_s/T_1 <1

$$T_S = S_{D1} / S_{DS}$$

$$R_d = \frac{1.5R}{\Omega_0}$$



Simplified Pushover Approaches: 2003 *NEHRP Provisions* (3)

- Member strengths need not be evaluated
- Component deformation acceptance based on laboratory tests
- Maximum story drift may be as high as 1.25 times standard limit
- Nonlinear Analysis must be Peer Reviewed

Simplified Pushover Approaches: FEMA 356*. (Also used in FEMA 350)

- Procedure presented in Chapter 3
- More detailed (thoughtful) treatment than in *NEHRP Recommended Provisions*

Principal Differences:

> Apply 25% of unreduced Gravity Load
> Use of two different lateral load patterns
> P-Delta effects included
> Consideration of Hysteretic Behavior

*** FEMA 273 in Prestandard Format**

Simplified Pushover Approaches: FEMA 356 (2)

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^{\,2}}{4\pi^2} g$$

Spectral Displacement

δ_t = Target Displacement

C_0 = Modification factor to relate roof displacement to first mode spectral displacement.

C_1 = Modification factor to relate expected maximum inelastic displacement to displacement calculated from elastic response (similar to NEHRP *Provisions* C_i)

Simplified Pushover Approaches: FEMA 356 (3)

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^{\ 2}}{4\pi^2} g$$

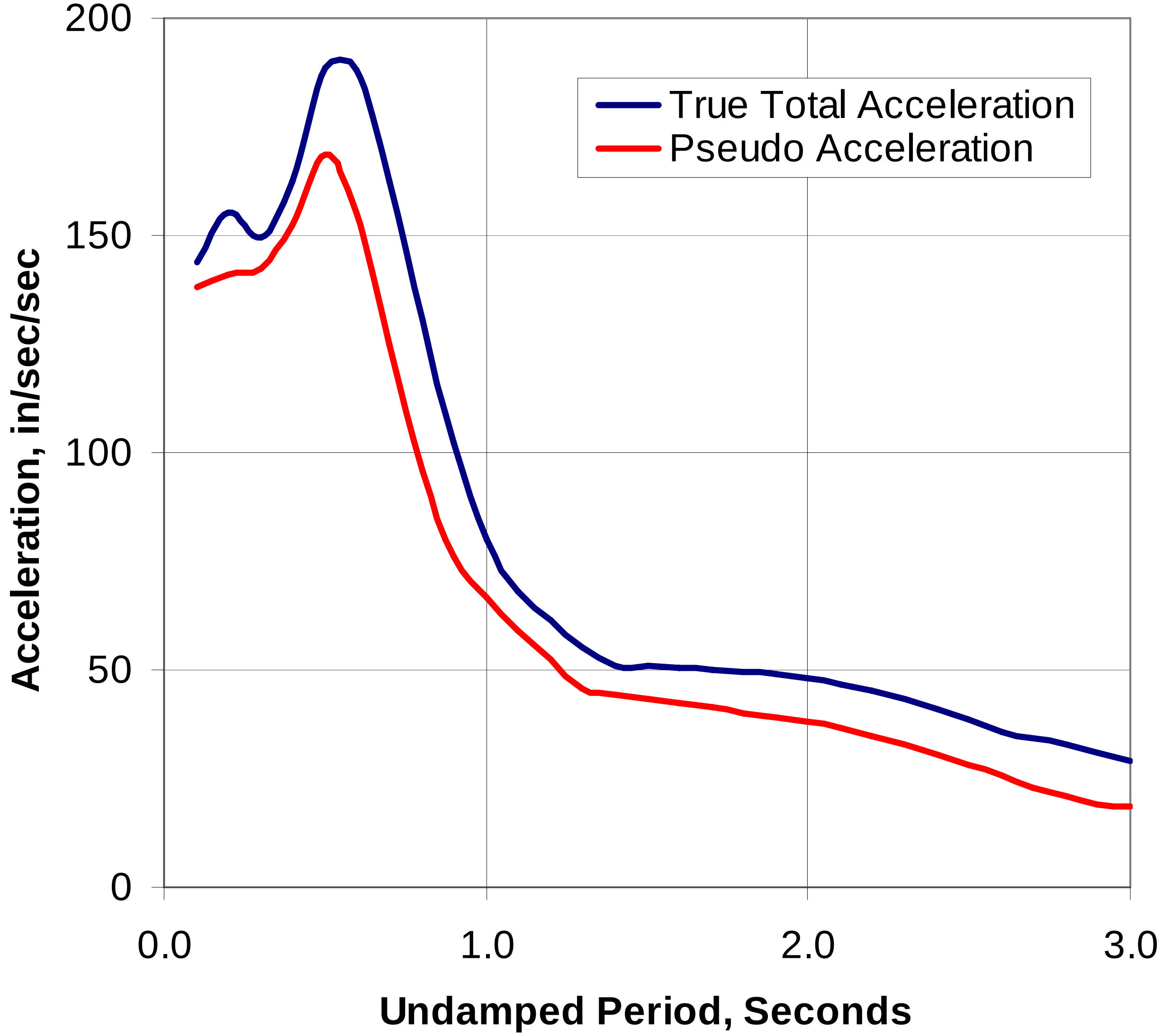
C_2 = Modification factor to represent effect of pinched hysteretic loop, stiffness degradation, and strength loss.

C_3 = Modification factor to represent increased displacements due to dynamic P-Delta effect

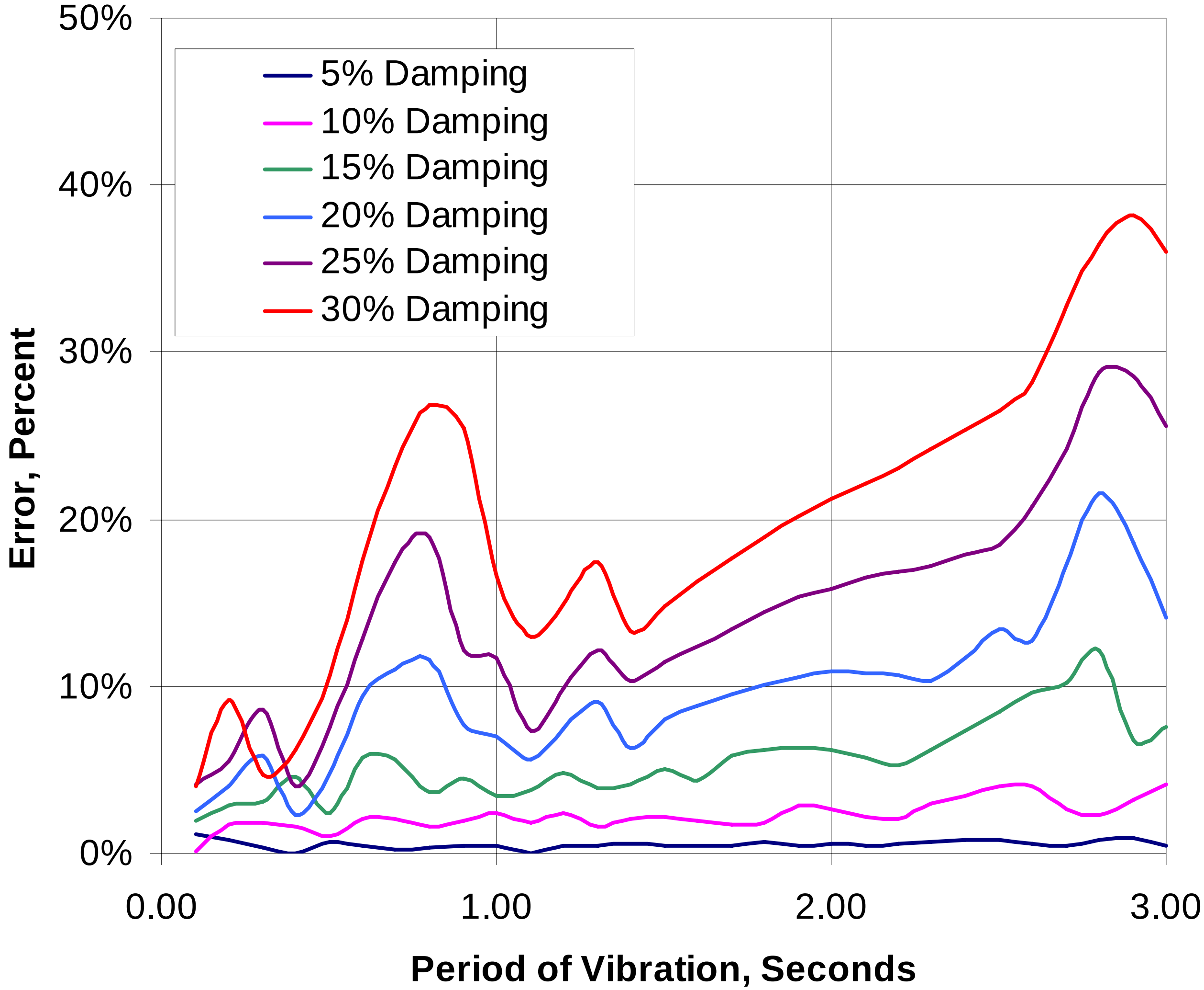
Discussion of Assumptions

1. Dynamic effects are ignored
2. Duration effects are ignored
3. Choice of lateral load pattern
4. Only first mode response included
5. Use of elastic response spectrum
6. Use of equivalent viscous damping
7. Modification of response spectrum for higher damping

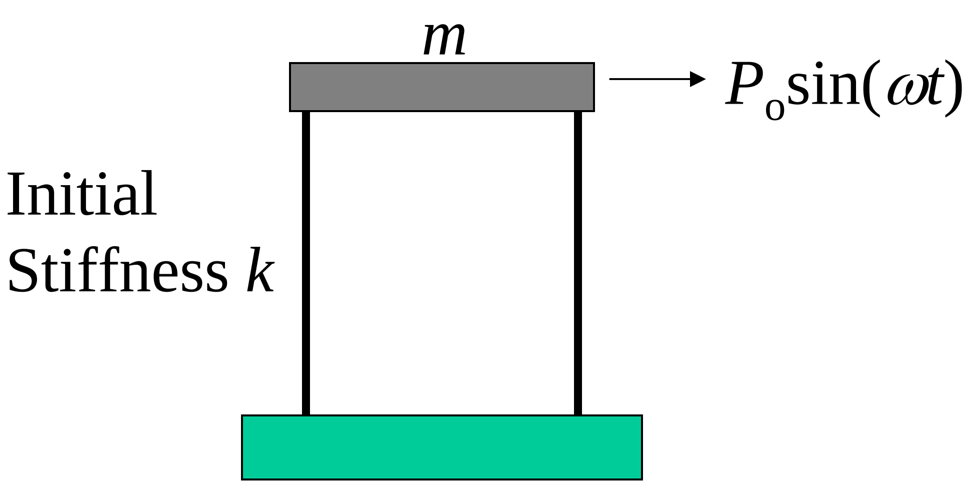
True Acceleration vs Pseudoacceleration 30% Critical Damping

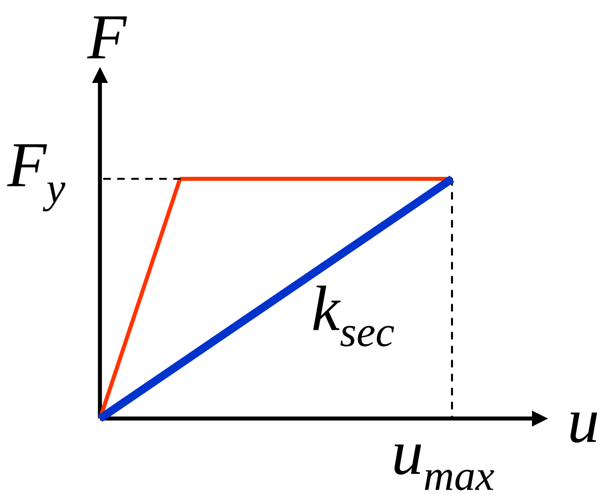


Relative Error Between True Acceleration and Pseudoacceleration



“Equivalent” Elastic System





Resonant Frequency:

$$\omega_{\sec} = \sqrt{\frac{k_{\sec}}{m}}$$

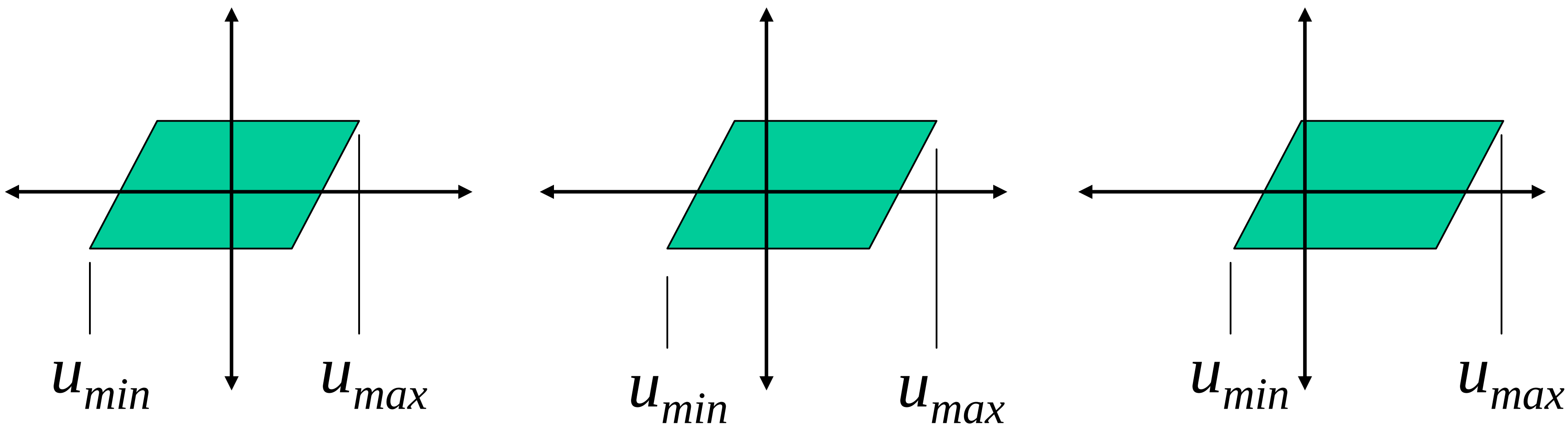
Maximum Steady State Resonant Response:

$$u_{\max} = \frac{k_{\sec}}{2\xi_{\sec} P_o}$$

Equivalent Damping:

$$\xi_{\sec} = 0.637\left(1 - \frac{u_y}{u_{\max}}\right)$$

These systems have the same hysteretic Energy Dissipation, the same AVERAGE (+/-) displacement, but considerably DIFFERENT maximum displacement.

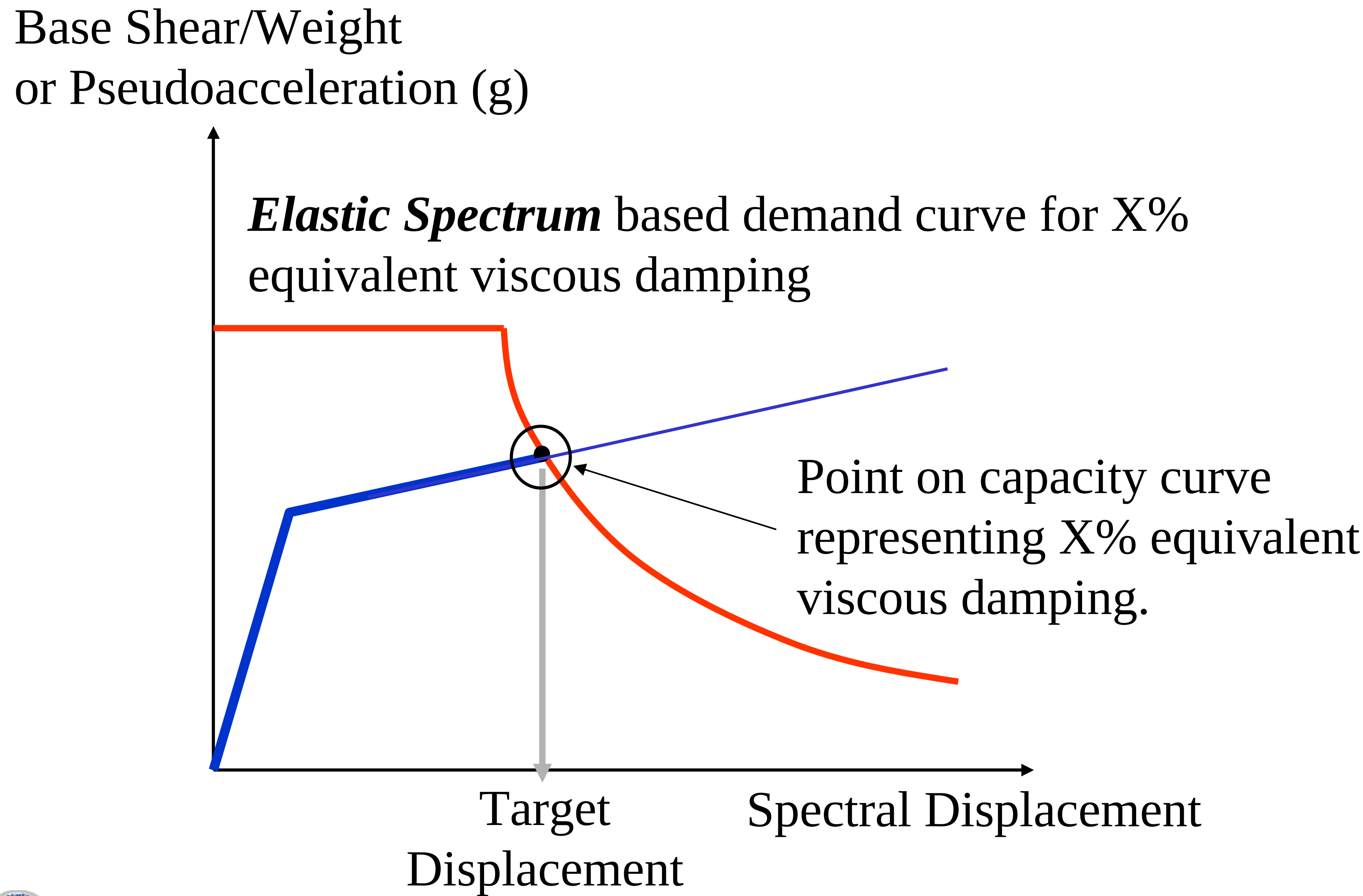


The equivalent viscous damping (see previous slide) is good at predicting the AVERAGE displacement, but CAN NOT predict the true maximum displacement.

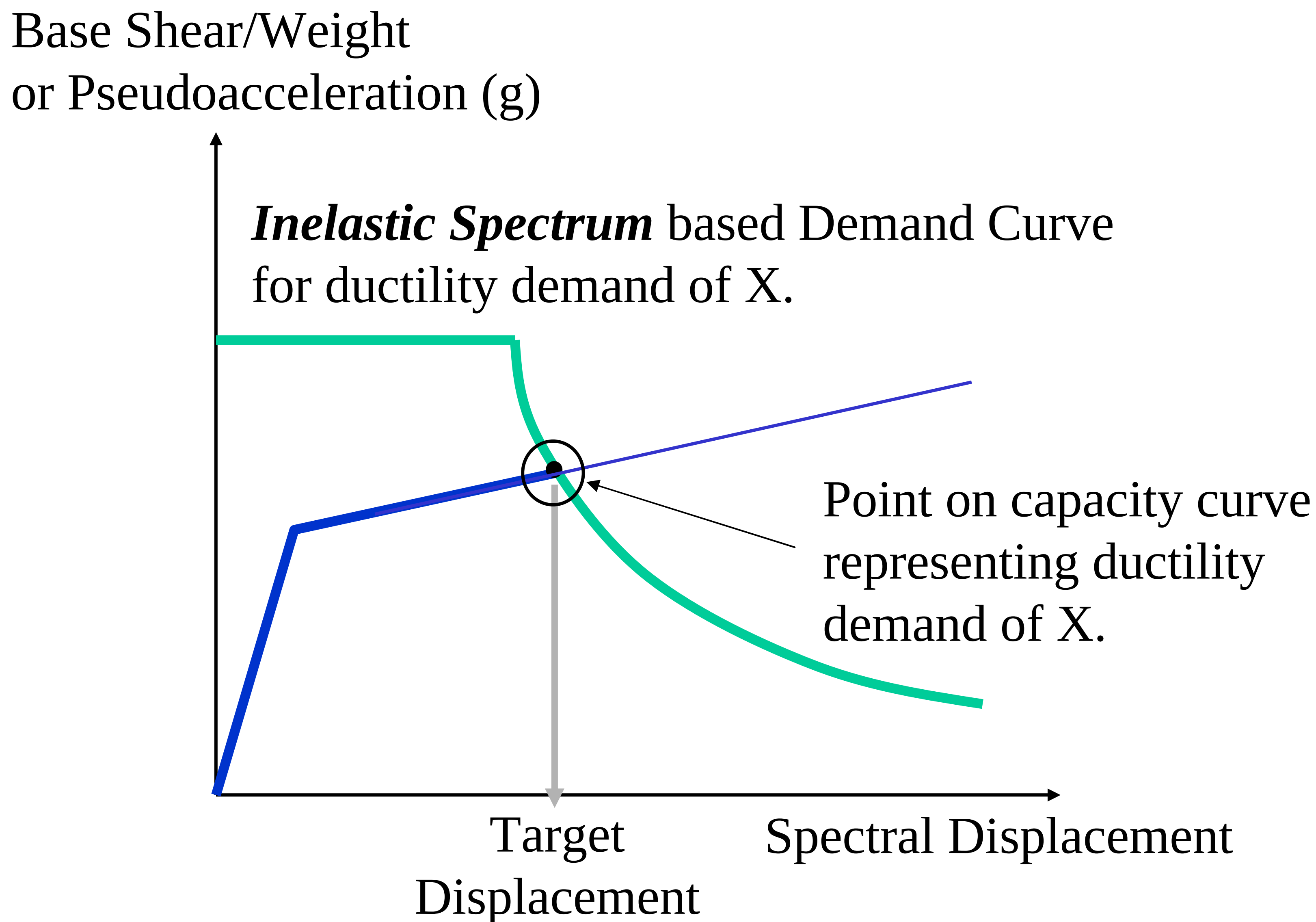
"Improved" Pushover Methods

- Use of Inelastic Response Spectrum
- Adaptive Load Patterns
- Use of SDOF Response History Analysis
- Inclusion of Higher Mode Effects

Elastic Spectrum Based Target Displacement



Inelastic Response Spectrum Based Target Displacement



Inelastic Spectrum Based Target Displacement

- Gives the same results as the equal displacement theory for (longer period) EPP systems
- When compared to inelastic response history analysis, the use of inelastic spectra gives better results than ATC 40 procedure.

Computing Target Displacements from Response History Analysis of SDOF Systems

- Method called "Uncoupled Modal Response History Analysis" (UMRHA) is described by Chopra and Goel. See, for example, Appendix A of PEER Report 2001/03, entitled *Modal Pushover Analysis Procedure to Estimate Seismic Demands for Buildings*.
- In the UMHRA method, the undamped mode shapes are used to determine a static load pattern for each mode.
- Using these static lateral loads, a series of pushover curves and corresponding bilinear capacity curves are obtained for the first few modes. This is done using the procedures described earlier for the ATC 40 approach.

Computing Target Displacements from Response History Analysis of SDOF Systems (2)

- Using an appropriate ground motion, a nonlinear dynamic response history analysis is computed for each modal bilinear system. This may be accomplished using NONLIN or NONLIN-Pro.
- The modal response histories are transformed to system coordinates and displacement (and deformation) response histories are obtained for each mode.
- The modal response histories are added algebraically to determine the final displacement (deformations). In the Modal Pushover approach, the individual response histories are combined using SRSS.

Computing Target Displacements from Response History Analysis of SDOF Systems (3)

- Results from such an analysis are detailed in PEER Report 2001/16, entitled *Statistics of SDF-System Estimate of Roof Displacement for Pushover Analysis of Buildings*.

Conclusions from above report (paraphrased by F. Charney):

- For larger ductility demands the SDOF method, using only the first mode, overestimates roof displacements and the bias increases for longer period buildings.

- For small ductility demand systems, the SDOF system, using only the first mode, underestimates displacement, and the bias increases for longer period systems.

Conclusions (continued)

- First mode SDOF estimates of roof displacements due to individual ground motions can be alarmingly small (as low as 0.31 to 0.82 times "exact") to surprisingly large (1.45 to 2.15 times exact).
- Errors increase when P-Delta effects are included. (Note: the method includes P-Delta effects only in the first mode).
- The large errors arise because for individual ground motions the first mode SDOF system may underestimate or overestimate the residual deformation due to yield-induced permanent drift.
- The error is not improved significantly by including higher mode contributions. However, the dispersion is reduced when elastic or nearly elastic systems are considered.

Computing Target Displacements from Response History Analysis of SDOF Systems

Problems with the method:

- **No rational basis**
- Does not include P-Delta effects in higher modes
- Can not consider differences in hysteretic behavior of individual components
- No reduction in effort compared to full time-history analysis
- Problem of ground motion selection and scaling still exists