Appendix R
COMBINED SAMPLING AND MODELING UNCERTAINTY

R.1 Methodology

Sources of uncertainty. As has been noted elsewhere, the total benefit of FEMA grants is uncertain. It was desired to quantify and combine all important sources of uncertainty. This information was then used to calculate two interesting parameters: (a) confidence bounds for the total benefit of FEMA grants for each hazard, and (b) the probability that the "true" benefits exceed the cost. By "confidence bounds" is meant upper and lower bounds between which the "true" total benefit lies with any given level of probability. The uncertainty in total benefit of FEMA grants results from two principle sources:

(1) *Sampling uncertainty*. Total benefits are uncertain because they are estimated from a sample (a subset) of FEMA grants, not the entire population of them.

(2) *Modeling uncertainty*. Total benefits are uncertain because a mathematical model of benefits has been created and applied, and that mathematical model has its own uncertain parameters.

Measures of uncertainty. Let X denote (uncertain) total benefit of FEMA grants. Let $x_{l,p}$ and $x_{u,p}$ denote the lower and upper bounds of X, respectively, that corresponding to probability p that total benefit lies between them. Further, let the confidence bounds be symmetric in that

$$\begin{aligned} p &\equiv P\left[x_{l,p} < X \le x_{u,p}\right] \\ P\left[x_{l,p} < X\right] &= P\left[X \le x_{u,p}\right] = \left(\frac{1+p}{2}\right) \end{aligned} \quad \text{(R-1)}$$

One can calculate the effect of each type of uncertainty and combine them into an overall estimate of the uncertainty of total benefit. To begin this process, it is reasonable to assume that the total must be greater than or equal to zero, i.e., that no mitigation actually has negative benefit. Without any additional knowledge, by information theory (Shannon and Weaver, 1963), the best assumption for the distribution of total benefit is the lognormal distribution, i.e.,

$$F_X\left(x\right) \equiv P\left[X \le x\right] = \Phi\left(\frac{\ln\left(x\right)-\lambda}{\beta}\right) \quad \text{(R-2)}$$

where x represents a particular value of X, $F_X(x)$ denotes the cumulative distribution function of X, P denotes probability, Φ denotes the cumulative standard normal distribution, and λ and β are parameters of the distribution, referred to as the logarithmic mean and logarithmic standard deviation. If C denotes the total cost of FEMA grants, then the probability that benefit exceeds cost is given by

$$\begin{aligned} P\left[X > C\right] &= 1 - F_X\left(C\right) \\ &= 1 - \Phi\left(\frac{\ln\left(C\right)-\lambda}{\beta}\right) \end{aligned} \quad \text{(R-3)}$$

and the confidence bounds $x_{l,p}$ and $x_{u,p}$ are given by

$$x_{l,p} = \exp\left(\Phi^{-1}\left(\frac{1-p}{2} \right)\beta + \lambda \right)$$
$$x_{l,p} = \exp\left(\Phi^{-1}\left(\frac{1+p}{2} \right)\beta + \lambda \right) \qquad \text{(R-4)}$$

where Φ^{-1} denotes the inverse cumulative standard normal distribution. Denoting the sample mean value of X by m_X, parameter λ is given by

$$\lambda = \ln\left(m_X\right) - 0.5\beta^2 \qquad \text{(R-5)}$$

Combining uncertainty. It is common to assume that sampling uncertainty is independent of modeling uncertainty, and that one can estimate β as

$$\beta = \sqrt{\beta_1^2 + \beta_2^2} \qquad \text{(R-6)}$$

where β_1 denotes the logarithmic standard deviation of X resulting from sampling uncertainty, and β_2 denotes the logarithmic standard deviation of X resulting from modeling uncertainty.

Sampling uncertainty. One can calculate β_1 as

$$\beta_1 = \sqrt{\ln\left(1 + \left(\frac{s_X}{m_X\sqrt{n}} \right)^2 \right)} \qquad \text{(R-7)}$$

where ln denotes the natural logarithm, s_X denotes the sample standard deviation of X and n denotes the sample size. If one knows m_X and the sample standard deviation and sample mean of benefit-cost ratio (s_{BCR} and m_{BCR}, respectively), it is straightforward to calculate s_X as

$$s_X = m_X \cdot \left(\frac{s_{BCR}}{m_{BCR}} \right) \qquad \text{(R-8)}$$

Modeling uncertainty. One can calculate β_2 as

$$\beta_2 = \sqrt{\ln\left(1 + \left(\frac{\sigma_X}{\mu_X} \right)^2 \right)} \qquad \text{(R-9)}$$

where σ_X denotes the standard deviation of X associated with modeling uncertainty, and μ_X denotes the mean value of X, considering modeling uncertainty.

R.2 Results

All the required parameters were available for these calculations. The values of C, m_X, s_{BCR}, and m_{BCR} are shown in Tables 6-1, 6-3, and 6-4. The values of n are not shown elsewhere, but were available from the sample data. The parameters σ_X and μ_X are presented in Section 6.5, the tornado-diagram analyses. Table R-1 presents the results for the symmetric 90% bounds of the total benefit of FEMA grants. Two interesting observations are apparent:

1. Modeling uncertainty dominates total uncertainty ($\beta_1 << \beta_2$, so $\beta \approx \beta_2$), so larger sample would not improve the accuracy of the estimated benefits.
2. The results reaffirm the observation that project mitigation grants produce benefits in excess of costs with high probability for all three hazards.
3.

Table R-1 Combined sample uncertainty and modeling uncertainty

Parameter of Interest	Projects			Source
	Earthquake	Wind	Flood	
Sample properties (*n*)	128	204	483	Sample data
Total cost of grants ($M) (*C*)	$ 867	$ 280	$ 2,204	Table 6-1
Total benefit of grants ($M) ($m_X$)	$ 1,194	$ 1,307	$ 11,172	Table 6-3
Total sample mean BCR (m_{BCR})	1.4	4.7	5.1	Table 6-4
Sampling uncertainty				
Sample standard deviation of BCR (s_{BCR})	1.3	7.0	1.1	Table 6-4
Standard deviation of benefit ($M) ($s_X$)	$ 1,157	$ 1,969	$ 2,424	Equation (R-8)
$_1$	0.09	0.11	0.01	Equation (R-7)
Modeling uncertainty				
Mean benefit of grants ($M) ($_X$)	$ 1,288	$ 1,308	$ 10,494	Section 6.5
Standard deviation of benefit ($M) ($_X$)	$ 468	$ 555	$ 3,778	Section 6.5
$_2$	0.35	0.41	0.35	Equation (R-9)
Total uncertainty				
	0.36	0.42	0.35	Equation (R-6)
	7.02	7.09	9.26	Equation (R-5)
Probability that benefit exceeds cost	76%	99.97%	99.9996%	Equation (R-3)
90-percent bounds of benefit of FEMA grants				
Lower-bound benefit ($M) ($x_{l,0.90}$)	$ 617	$ 600	$ 5,918	Equation (R-4)
Upper-bound benefit ($M) ($x_{u,0.90}$)	$ 2,029	$ 2,389	$ 18,670	Equation (R-4)