Appendix N
FOUR METHODS TO SELECT SAMPLE AND SCALE-UP BENEFIT

N.1 Summary

This appendix documents four methods to select a sample of size N = 25 from a population of mitigation efforts and to calculate total stratum benefit. In all four methods, stratified sampling is used to ensure sampling of the tails of the distribution of approved net eligible project cost (referred to hereafter as cost). In summary, the methods are as follows.

Method 1**:** mitigation efforts are selected for sampling with equal probability, using strata of equal sizes, and population benefit is estimated as the sum of the sample benefits times L/N, where L is the number of mitigation efforts in the population.

Method 1b**:** mitigation efforts are selected for sampling with equal probability, using strata of equal sizes, and population benefit is estimated as $B' = C^*\mu_{bcr}$, where μ_{bcr} is the sample-average benefit-cost ratio.

Method 2**:** mitigation efforts are selected with probability in proportion to their cost, using strata of equal cumulative cost, and population benefit is estimated as the sum of the sample benefits times C/c, where C is the population cost and c is the cost of the sample.

Method 3**:** mitigation efforts are selected with probability in proportion to their cost, using strata of equal cumulative cost, and population benefit is estimated as $B' = C^*\mu_{bcr}$, where μ_{bcr} is the sample-average benefit-cost ratio.

Let:

L = population size (number of mitigation efforts in the population)
N = sample size (number of mitigations in the sample)
c_i = cost of mitigation i.
bcr_i = benefit-cost-ratio of mitigation i
b_i = benefit of mitigation $i = c_i bcr_i$
C = the total cost of all mitigations
B' = estimated benefit of population based on sample

$= (L/N)^*\Sigma_N b_i$ method 1 (N-1)

$= (C/\Sigma_N c_i)^*\Sigma_N b_i$ method 2 (N-2)

$= C^*\Sigma_N bcr_i/N$ methods 1b and 3 (N-3)

B = true population benefit = $\Sigma_L b_i$
ε = relative error of benefit estimate

$= (B' - B)/B$ (N-4)

μ_ε = mean relative error of benefit estimate
σ_ε = standard deviation of relative error of benefit estimate

Two reasonable criteria for accepting a sampling method are: (1) it produces an unbiased estimate of total benefit, i.e., $\mu_\varepsilon \approx 0.0$, and (2) it produces a small uncertainty in the estimate of total benefit, i.e., σ_ε is small. The criterion for acceptable σ_ε is that the uncertainty is small enough that one can answer with 90% confidence whether FEMA grants have been cost effective, i.e., either:

$$B'*(1 - 1.28\sigma_\varepsilon)/C > 1.0 \text{ or equivalently } (1 - C/B')/1.28 > \sigma_\varepsilon$$

or

$$B'*(1 + 1.28\sigma_\varepsilon)/C < 1.0$$

In the former case, one can say with 90% confidence that the population of mitigation efforts within the stratum is cost-effective; in the latter, one can say with 90% confidence, the population of mitigation efforts within the stratum is not cost-effective. Both assume normality of B', an unbiased estimate of B, and ignore error in the estimation of benefit for an individual mitigation effort, b_i. For an unbiased estimator, $E[B'] = B = \$5.57*10^9$ and $C = \$2.36*10^9$. An acceptable sampling approach must therefore have $\sigma_\varepsilon < (1 - C/B)/1.28$, or $\sigma_\varepsilon < 0.45$. Only method 3 passes this criterion.

Explanations of the mechanics of these selection and benefit-calculation procedures follow.

N.2 Method 1

This method applies an equal probability of a grant being sampled, and benefits are scaled up in proportion to number of grants sampled. Method 1 is performed as follows.

1. Stratify project-type mitigation activities by peril (earthquake, wind, flood) and hazard level. The following steps are repeated for each stratum.
2. Select N, the number of samples per stratum. In this project, $N = 25$.
3. Sort the population in increasing c_i.
4. Divide the stratum population in N contiguous bins of increasing cost, with an equal number n of projects in each bin (±1, to account for a stratum population that is not an integer multiple of N).
5. Assign a random number u, uniformly distributed between 0 and 1, to each mitigation effort.
6. Re-sort projects by increasing bin number and then by increasing u within the bin.
7. Select from each bin the project with the lowest value of u. The result is N randomly selected projects that nonetheless span the range of project costs.
8. Calculate the benefit for each mitigation effort in the sample, b_i: $i = 1, 2, \ldots N$, where i now indexes mitigation efforts in the sample.
9. Calculate B' per Equation N-1.

N.3 Method 1b

This method applies an equal probability of a grant being sampled, and scales up benefits by averaging sample benefit-cost ratio (BCR). Method 1b is performed as shown under Method 1, except that B' is calculated per Equation N-3.

N.4 Method 2

This method applies probability of a grant being selected in proportion to its cost, and scales up benefit in proportion to the cost of sampled grants. It works as follows.

1. Stratify project-type mitigation activities by peril (earthquake, wind, flood) and hazard level. The following steps are repeated for each stratum.
2. Select N, the number of samples per stratum. In this project, $N = 25$.
3. Sort the population in increasing c_i.
4. For each mitigation effort i, calculate the cumulative fraction of total cost, $F_C(c_i) = \Sigma_{j=0..i}c_j$. Divide the population in N contiguous bins of increasing project cost, with equal total bin cost, i.e., bin k includes mitigation efforts p, p+1, … q such that $\Sigma_{j=p..q}c_j = C/N$.
5. Assign a random number u, uniformly distributed between 0 and 1, to each mitigation effort.
6. Select from each bin the project with the lowest value of u. The result is N randomly selected projects that both span the range of cost and place more emphasis on costlier projects.
7. Calculate the benefit for each mitigation effort in the sample, $b_i = bcr_i*c_i$, $i = 1, 2, \ldots N$, where i indexes mitigation efforts in the sample.
8. Calculate B' per Equation N-2.

N.5 Method 3

In this method, the probability of sample selection is proportional to its cost, and the benefit is scaled up by calculating the sample-average BCR and applying this BCR to the stratum.

1. Stratify project-type mitigation activities by peril (earthquake, wind, flood) and hazard level. The following steps are repeated for each stratum.
2. Select N, the number of samples per stratum. In this project, $N = 25$.
3. Sort the population in increasing c_i.
4. For each mitigation effort i, calculate the cumulative fraction of total cost, $F_C(c_i) = \Sigma_{j=0..i}c_j$. Divide the population in N contiguous bins of increasing project cost, with equal total bin cost, i.e., bin k includes mitigation efforts p, p+1, … q such that $\Sigma_{j=p..q}c_j = C/N$.
5. Assign a random number u, uniformly distributed between 0 and 1, to each mitigation effort.

6. Select from each bin the project with the lowest value of u. The result is N randomly selected projects that both span the range of cost and place more emphasis on costlier projects.
7. Calculate mean benefit-cost ratio for the sample, μ_{bcr}: $= \Sigma_N bcr_i/N$, $i = 1, 2, \ldots N$, where i indexes mitigation efforts in the sample.
8. Calculate B' per Equation N-3.

N.6 Tests of Method 1

Simulated population. A simulated (hypothetical) population of $L = 1000$ mitigation efforts was created whose cost distribution match that of the FEMA grants, i.e., lognormal with median cost = \$732,000 and logarithmic standard deviation = 1.80. It was necessary to assign a value of benefit to each mitigation effort. To do this, the benefit-cost ratios (BCRs) in the NEMIS grant database were examined, and those with project cost (denoted by C) > 1 and BCR > 1 extracted. Of the extracted grants, it is found that the average estimated BCR is 10.3, with a logarithmic standard deviation of 0.87. Project cost and BCR appear to be uncorrelated, either for the population (correlation coefficient $\rho = -0.0097$, $N = 3176$), wind mitigation grants ($\rho = -0.025$) or flood mitigation grants ($\rho = -0.024$); a modest negative correlation exists for earthquake mitigation grants ($\rho = -0.10$). A lognormal distribution was assigned to BCR using the statistics quoted above and BCRs were simulated for each mitigation grant in the simulated population.

Testing for bias. The hypothetical population was grouped into $N = 25$ strata of $M = L/N = 40$ samples per stratum, with the substrata grouped by increasing cost, per the sampling approach described above. The Excel add-in "Insight.xla" (see www.duxbury.com) was used to create Q = 1000 simulated sample sets of 25 mitigation efforts, each time calculating the actual population benefit $B = \Sigma_L b$ and the estimated benefit $B' = M\Sigma_N b$, and calculated the error per Equation N-4. There is one value of ε for each sample set, i.e., there are $Q = 1000$ samples of ε. One can calculate a mean bias as $\mu_\varepsilon = 1/Q^*\Sigma_Q\varepsilon$. A value of $\mu_\varepsilon \neq 0.0$ indicates a bias. In these expressions, b is the benefit from one mitigation effort, Σ_L indicates the sum over the population of L mitigation efforts, $\Sigma_.$ indicates the sum over the sample of N mitigation efforts, Σ_Q indicates a sum over Q sample sets, B indicates the "true" total population benefit, and B' indicates the estimated population benefit extrapolated from the sample.

Observations. This simulation approach produces an unbiased estimate of benefit. Using Q = 1000 simulation produces an estimated mean error, $\mu_\varepsilon = -0.022$, and an estimated standard deviation of error $\sigma_\varepsilon = 0.69$, which is too large. One observes an unbiased estimate if BCR is assumed to be a constant value (BCR = 2 produces $\mu_\varepsilon = 0.0014$), if BCR is assumed to increase linearly with cost (BCR = 1 + C/100 produces $\mu_\varepsilon = 0.013$), to linearly decrease with cost (BCR = 5 – C/100 produces $\mu_\varepsilon = 0.013$) or to be quadratic with cost (BCR = $1 + (C/100)^2$ produces $\mu_\varepsilon =$ 0.039).

Testing using the NEMIS population. The bias test was repeated using a subset of the NEMIS portfolio: all those in-scope mitigation efforts whose $C > 1$ and whose BCR > 1. The subset includes $L = 3176$ mitigation efforts. These were stratified into $N = 25$ strata of $M = 127$ efforts each (the first stratum had $M = 128$). The Microsoft Excel add-in Insight.xla was used to create

1000 sample sets of 25 mitigation efforts, each time calculating the actual population benefit and the estimated benefit $B' = (L/N)\Sigma b$, and calculating the error per Equation N-4. For the "actual" population benefit, the estimated BCRs from the NEMIS database were used: $B = \Sigma_L b = \Sigma_L bcr_i * c_i$. Again using $Q = 1000$, it is found that $\mu_\varepsilon = 0.00058$, which suggests no bias, and a standard deviation of error, $\sigma_\varepsilon = 0.55$, which is approximately equal to that obtained using the simulated portfolio. A test using Method 1b produces a biased and highly uncertain estimate: $\mu_\varepsilon = 0.82$ and $\sigma_\varepsilon = 1.56$.

Method 1 has unacceptably high uncertainty. Method 1b has unacceptable bias and uncertainty.

N.7 Tests of Method 2

This approach was tested once using the simulated population (with random BCR distributed the same as FEMA's estimate shown in the NEMIS population) and once using the NEMIS population. Using the simulated population, this approach produces an unbiased estimate of total benefit, with better accuracy than Method 1: in $Q = 1000$ simulation, one finds $\mu_\varepsilon = -0.010$, and standard deviation of error, $\sigma_\varepsilon = 0.17$. Comparing this $\sigma_\varepsilon = 0.17$ with 0.69 using Method 1 suggests that Method 2 produces a much more-accurate estimate of total population benefit.

However, using the NEMIS population and NEMIS benefits, this method underestimates the population benefit, albeit with very low variability: $\mu_\varepsilon = -0.40$ and $\sigma_\varepsilon = 0.05$. The reason appears to be the slight negative trend of BCR with cost; although $\rho_{c,bcr} = -0.0097$, the trend is strong enough to produce a consistent under-estimate of benefit. That is, benefit accrues disproportionately from smaller projects. Again, this test assumes that the existing FEMA estimates of benefit are unbiased with respect to cost, i.e., that the "true" BCR follows the same trend with cost as does the BCR estimated by FEMA.

Method 2 has unacceptably high bias.

N.8 Tests of Method 3

Method 3 was tested both with the synthetic population and the NEMIS population. The former produced an unbiased estimate of *B*, with $\mu_\varepsilon = 0.0078$ and $\sigma_\varepsilon = 0.13$; the latter a biased estimate: $\mu_\varepsilon = 0.42$ and $\sigma_\varepsilon = 5.18$. The reason is that there are four mitigation efforts in the NEMIS portfolio with $bcr \approx 3300$ and one with $bcr \approx 6200$. They have low cost, so their effect is small under method 2, but method 3 is sensitive to them. When these are eliminated from the population, $\mu_\varepsilon = 0.023$ and $\sigma_\varepsilon = 0.39$, i.e., an unbiased estimate of benefit with a moderate uncertainty. (The previous methods were also checked after censoring these high BCRs; this approach makes too little difference to accept Methods 1, 1b, or 2.)

Method 3 has an acceptable uncertainty, as long as one assumes that samples of BCR > 1000 are erroneous.